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Focus on Fractions
Building Fraction Sense
Why Not?

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All students should leave elementary and middle school with a strong sense of number

What does that mean?

How do we do that?

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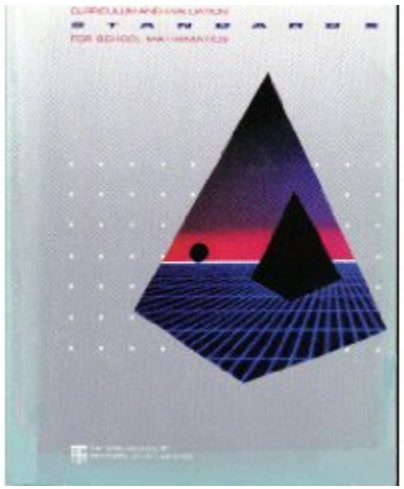
Number Sense - History

- Number Meaning
- Relationships
- Magnitude
- Operation Sense
- Real Life Number Sense - Applications



Howden, 1989

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Making Sense of Numbers…

1. Ability to compose and decompose numbers…
2. Ability to recognize the relative magnitude of numbers – including comparing and ordering.
3. Ability to deal with the absolute magnitude of numbers – realizing, for instance there are far fewer than 500 people in this session!
4. Ability to use benchmarks.
5. Ability to link numeration, operation, and relation symbols in meaningful ways.
6. Understanding the effects of operations on numbers.
7. The ability to perform mental computation through invented strategies that take advantages of numerical and operational properties.
8. Being able to use numbers flexibly to estimate numerical answers to computations, and to recognize when an estimate is appropriate.
9. A disposition towards making sense of numbers.

"It is possible to have good number sense for whole numbers, but not for fractions…"

Sowder, J. and Schappelle, Eds., 1989

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And more recently…

- In its most fundamental form, number sense entails an ability to immediately identify the numerical value associated with small quantities;
- …this more highly developed form of number sense should extend to numbers written in fraction, decimal, and exponential forms.
- …poor number sense interferes with learning algorithms and number facts and prevents use of strategies to verify if solutions to problems are reasonable.

NMAP, page 27

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How did this get started…

Fraction issues…

- Conceptual Knowledge and Skills
- Learning Processes
- Assessment
- Survey of Algebra Teachers

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Do you have a sense of number?

- Is 5/6 > 7/8? How do you know?
- What is 0.8 x 0.9?
- If the restaurant bill was $119.23, how much of a tip should you leave?
- “We need 110% effort in the 2nd half, now let’s go…”
- If a 10-year old is 5’ tall, how tall will the child be at age 20?

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Policy and Political Issues

- Where and how does this fit in with your state standards, the coming "common core" standards?
- This is so much more than whole numbers!

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Common Core Standards Issues

Grade 1 and 2

- Nope

Grade 3

- Unit Fractions as a point on a number line;
- Equivalent fractions;
- Whole numbers expressed as fractions;
- Fractions as a whole decomposed into parts;
- Compare and order fractions with equal numerators or denominators.

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Common Core Standards Issues

Grade 4

- Addition and subtraction of fractions;
- Understand the meaning of multiplication of fractions;
- Relate decimals and fractions;
- Compare and order decimals – like denominators.

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Common Core Standards Issues

Grade 5

- Read, write and compare decimals;
- Round decimals to the nearest whole number;
- Write fractions in decimal notation;
- Add, subtract, multiply, and divide decimals;
- Solve word problems involving operations with decimals;

Grade 5 (cont.)

- Understand fraction equivalence;
- Compare and order fractions with like and unlike denominators;
- Adding and subtracting fractions with like and unlike denominators;
- Understand multiplication of fractions;
- Understand division of unit fractions;
- Calculate products of fractions and quotients of unit fractions;
- Understand mixed numbers.

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Common Core Standards Issues

Grade 6

- Understand the concept of ratio;
- Equivalent ratios;
- Unit rate;
- Understand division of fractions via unknown factor multiplication;
- System of rational numbers.

Grade 7

- Form ratios;
- Compute unit rates and solve problems involving proportion;
- Plot proportional relationships on a coordinate plane;
- Understand %
- Find a percentage of a quantity;
- Solve multistep percent problems.
- Complex fractions;
- Operations with rational numbers (positive and negative).

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Common Core Standards Issues

another look

- **Number – Fractions**
  - Grade 3: representations, quantities
  - Grade 4: operations; decimals concepts
  - Grade 5: equivalence; operations
- **Ratio and Proportional Relationships**
  - Grade 6: ratios, unit rates
  - Grade 7: analyzing proportional relationships, percent
- **The Number System**
  - Grade 6: operations on fractions, system of rationals
  - Grade 7: system of rationals

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So, what's different…

- Chunking of fractions in grades 3-5 at the elementary level;
- Need for parallel development with decimals;
- Need to include benchmark percents earlier;
- Wonder about early fraction work – informal?

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What about research?

"…has a terrible time with basic skills. I mean, if we ever do anything with fractions, she's lost."*

Usher, …Middle School Students' Self-Efficacy in Mathematics, AERJ, 2009

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2nd Handbook Research on Mathematics Teaching & Learning

- Whole Number Concepts and Operations
  - Citations: 334
- Rational Numbers and Proportional Reasoning
  - Citations: 140
- In the 2000's: only 9 citations;
  - 109 in Whole Number Concepts and Operations
  - 1/12th

NCTM, 2007

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Hmm

- In my school, teachers can choose from fraction bars, fraction circles, fraction pieces that snap together and other concrete materials.
- "I feel my rationale for choosing a manipulative for instruction is selfish – it should not be about me, it should be about the kids."
- What fraction representation would best enhance student achievement at the 5th grade level?

Erin Prajzner, April 12, 2010

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IES* Practice Guide

- The research base for the guide was identified through a comprehensive search for studies over the past 20 years that evaluated teaching and learning about fractions.
- The process yielded more than 3,000 citations. Of these, 132 met the What Works Clearinghouse criteria for review (4%), and 33 met the causal validity standards of the WWC (1+%).

*Institute of Education Sciences

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- Fractions are a major area of study in upper elementary school mathematics. It is time to shift the emphasis and redefine the goal of fraction instruction from learning computation rules to developing fraction operation sense (Huinker, 2002).

- Do we do this?

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Number Meaning

$\frac{3}{10} = .3$

$\frac{17}{100} = .17$

$\frac{5}{100} = .05$

$\frac{323}{1000} = .323$

$\frac{47}{1000} = .047$

$\frac{9}{1000} = .009$

And for fractions…

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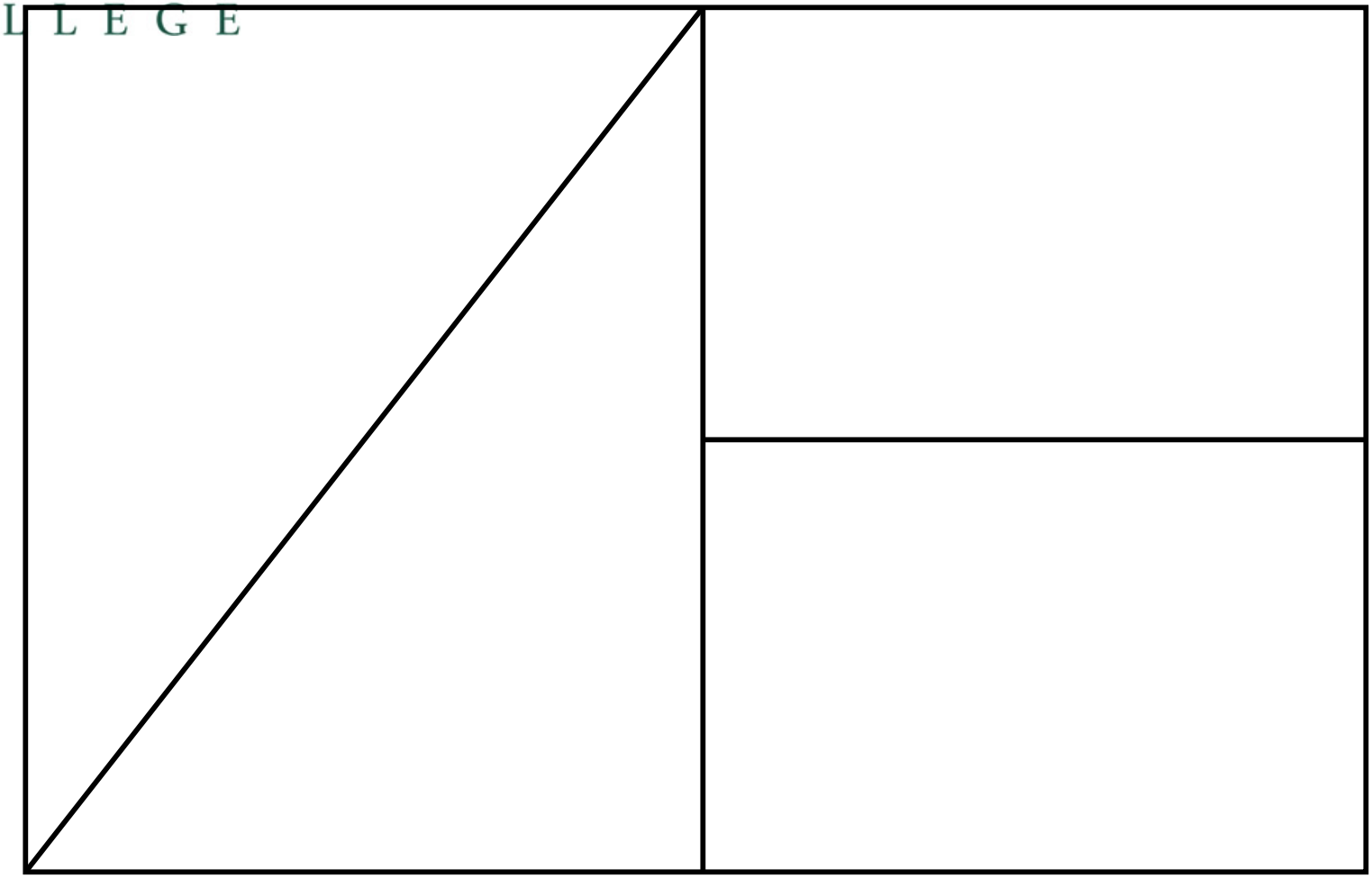

When the cake is cut, size of each piece?

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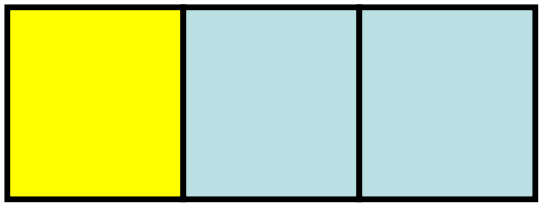
Fraction beginnings…

- Which one is larger, 1/2 or 1/3?



"the size of the fractional part is relative to the size of the whole…" (NCTM, 2006

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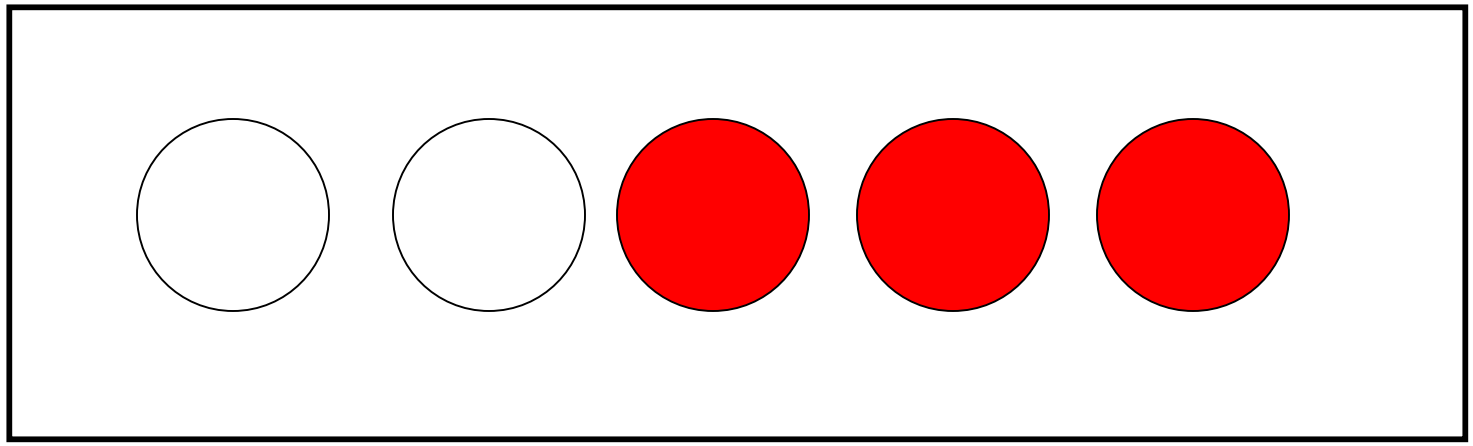

Could the drawing above represent:

a. 5/3 of something
b. 3/5 of something
c. 1 ÷ 3/5 = 1 2/3
d. 5 ÷ 3 = 1 2/3

Thompson and Saldanha, NCTM, 2003, page 107

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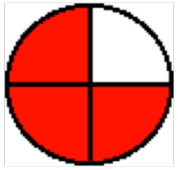
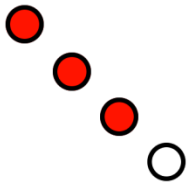
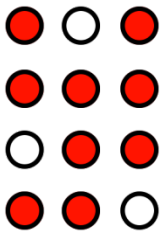
Thinking about ¾…

a) 

b) 

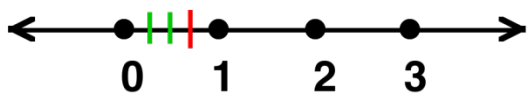
c) 

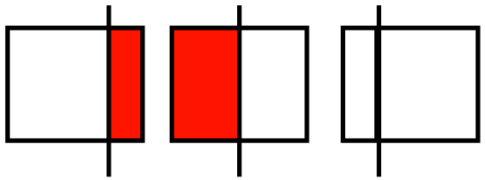
d) How many 4's are there in 3?

e) 18 crayons out of a box of 24

f) .75

g) I want to share 3 bottles of soda equally among 4 people. How much will each person get?

h) 

i) 

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Build on students' informal understanding of sharing and proportionality to develop initial fraction concepts.

IES – Practice Guide - Fractions, 2010

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Introducing fractions using sharing

- Four children want to share 10 cookies. Each child gets the same amount. How many cookies does each child get?
- Why?
  - Sharing is intuitive
  - Solution combines wholes and fractions
  - Sharing and repeated halving

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- How can we share **eleven** hoagies (aka subs) among four people?
- How can we share **eleven** hoagies (aka subs) among five people?

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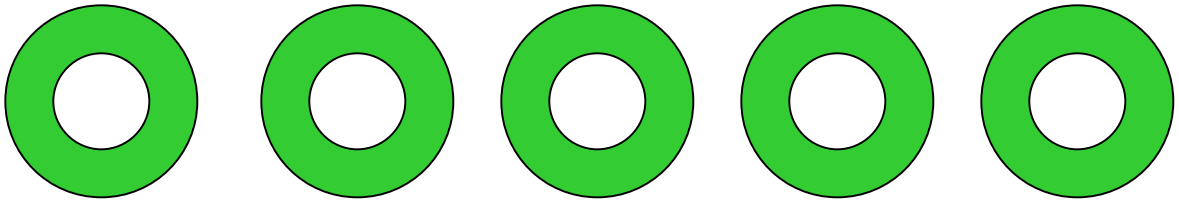
“How can I show 6 of 5, when I only have 5?”



Sometimes it’s language…

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How about if we have six people and we need to share 5 cookies?*

Division involving equal shares is a process that many understand intuitively.

*food seems to work – a lot!

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Sharing and Dividing

- 5 want to share 11
- 5 want to share 4
- 10 want to share 6

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Sense Making – I think…

- Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers.

- Use number lines as AN important representational tool in teaching this and other fraction concepts from the early grades onward.

IES – Practice Guide - Fractions, 2010

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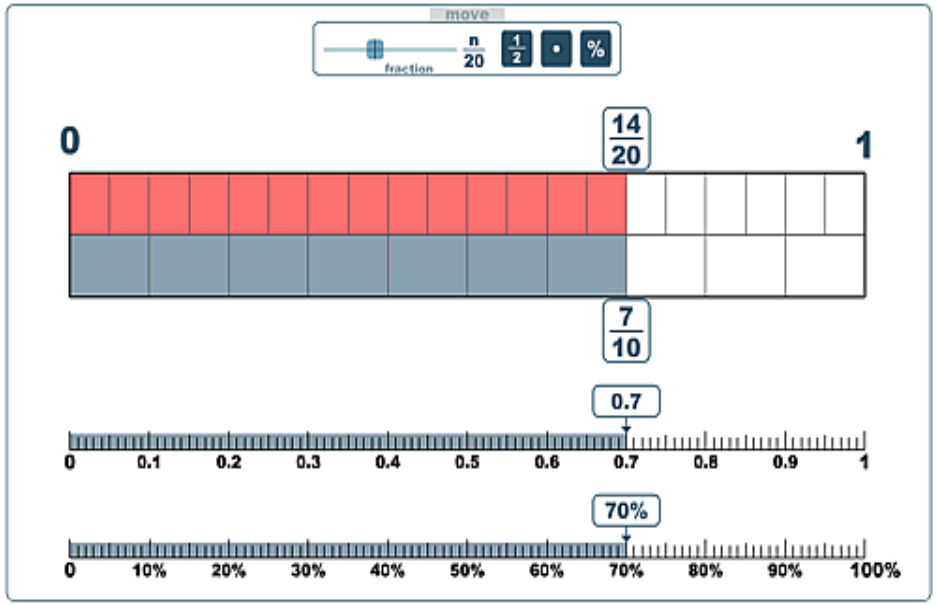
Name…

- A fraction between:
  - 0 and 1
  - ¼ and ½
  - 1/3 and 2/3
  - 5/6 and 1
  - 0 and 1/8
- A decimal between:
  - 1.1 and 1.2
  - 1.10 and 1.11
- …



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Math Wall Activities

2%

100%

3/4

3.11

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Fraction Sorting

- Sort the fractions below as near: 0, ½, or 1

4/7	1/7	8/9	3/5
2/3	1/10	4/8	6/11
4/5	2/12	9/12	5/12
1/8	3/8	4/9	7/14

- What's alike about all fractions near 1? Near 0?

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Ballparking

- About, around, close to:

  ½

  ¼

  25%

  10%

  1/3

  2/3

  75%

  200%

1. **Provide fraction or % and have students provide one close to that amount.**
2. **Provide a context and have them write about that amount/context using one of the ballpark amounts.**

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True or False – 5/6

- More than ½?
- Less than 0.75?
- More than 80%?
- The ratio of boys to girls in our class?
- Between ½ and 1?
- Between ¾ and 1?
- Less than 1

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$\frac{3}{4}$

- What happens to the value of the fraction if the numerator is increased by 1?
- What happens to the value of the fraction if the denominator is decreased by 1?
- What happens to the value of the fraction if the denominator is increased?

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Ordering Fractions

Write these fractions in order from least to greatest. Tell how you decided.

- 5/3 5/6 5/5 5/4 5/8
- 7/8 2/8 10/8 3/8 1/8

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- A student said that 3/4 and 5/6 are the same size because they both have one part missing: 3 is one less than 4 and 5 is one less than 6.

- Agree? Why or why not? How can you show the difference?

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Decimals

Order the following:

7.1

0.71

0.711

71

71.1

711

71.11

What helps you in ordering the decimals?

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Use Percent – Don't Wait!

- Put 2/3; 0.5 and 3/4 in order from smallest to largest.
- It's easy, 0.5 is 50% and 2/3 is 66%, and so it goes first 0.5, then 2/3 and then ¾ because that's 75%.*

*response by Andy in New Approaches to Teaching the Rational Number System by Joan Moss in How Students Learn: Mathematics in the Classroom, NRC, 2005.

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Percent Benchmarks

0%

100% **50%** **< 10%**

~25% **~75%** **~90%**

> 50% **< 50%**

- Lefthanders in the room or class
- Once lived in **New Jersey**
- Been involved in education > 10 years
- People who were born in **California**

Reys, et al, 1991

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You can't make this stuff up!

- The weather reporter on WCRB (a Boston radio station) said there was a 30% chance of rain. The host of the show asked what that meant. The weather reporter said ``It will rain on 30% of the state." ``What are the chances of getting wet if you are in that 30% of the state?" ``100%."

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Help students understand why procedures for computations with fractions make sense.

IES Practice Guide – Fractions, 2010

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More than or less than ONE

- 1/12 + 2/3
- 5/6 + 1/3
- ½ + ¼
- 1 ½ - 7/8
- 2/3 x 2/3
- 5/6 x 7/8
- 4/5 ÷ 2/3
- 9/10 – 1/12

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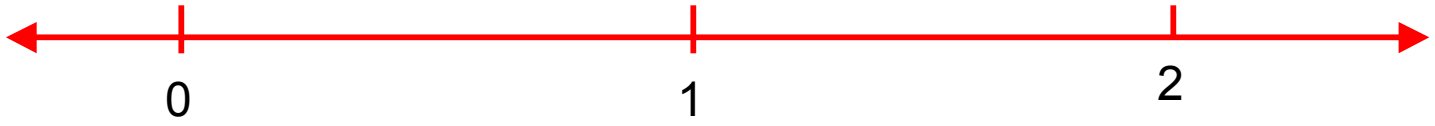
- Tell me about where 2/3 + 1/6 would be on this number line (Cramer, Henry, 2002).



Sense Making:

“2/3 is almost 1, 1/6 is a bit more, but the sum is < 1”

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7/8 – 1/8 = ?

- Interviewer: Melanie these two circles represent pies that were each cut into eight pieces for a party. This pie on the left had seven pieces eaten from it. How much pie is left there?
- **Melanie:** ***One-eighth, writes 1/8*****.**
- Interviewer: The pie on the right had three pieces eaten from it. How much is left of that pie?
- **Melanie:** ***Five-eighths, writes 5/8.***
- Interviewer: If you put those two together, how much of a pie is left?
- **Melanie:** ***Six-eighths, writes 6/8.***
- Interviewer: Could you write a number sentence to show what you just did?
- **Melanie:** ***Writes 1/8 + 5/8 = 6/16.***
- Interviewer: That's not the same as you told me before. Is that OK?
- **Melanie:** ***Yes, this is the answer you get when you add fractions.***

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What Happens Here?

- 1/2 x 3/4 **< or >** 3/4
- 3/4 x 1/2 **< or >** 1/2

- 1/2 ÷ 3/4 **< or >** 1/2
- 3/4 ÷ 1/2 **< or >** 3/4

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Now what?

- There are 25 students in our class. Each student will get ¼ of a pizza. Your job is to find out how many pizzas we should order. Be sure to show your work.

- How many pizzas should we order?

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Not Pizza

- Each tomato pie has ¼ cup of cheese sprinkled on top. Tianne is making 12 tomato pies for her extended family. How much cheese will she need?
- 12 x ¼ = ?

- Each tomato pie has ¼ of a cup of cheese sprinkled on top. If Tianne has 4 cups of cheese, on how many tomato pies can she sprinkle cheese?
- 4 ÷ ¼ ? or ¼ x ? = 4

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- How might you represent 7 x 2/3 and would you think of 2/3 x 7 differently?

- If you shared 7 doughnuts among 3 people, how could you use this to help determine 2/3 x 7?

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At Issue - Decimals

- Why start so early?
- Understanding of decimals draws on the understanding of fractions.
- 0.8 x 0.9 – think about 8/10 x 9/10

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The quotient is 0.5, what is the divisor and dividend?

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Decimals - What Happens?

Number	x 0.05	x 0.48	x 0.9
100			
60			
24			
?			

- In general, what happens when you multiply a whole number by: 0.05; 0.48; 0.9?
- Begin thinking of 0.05 as 5% or nickel:dollar, etc.

Reys (ed.), 1991

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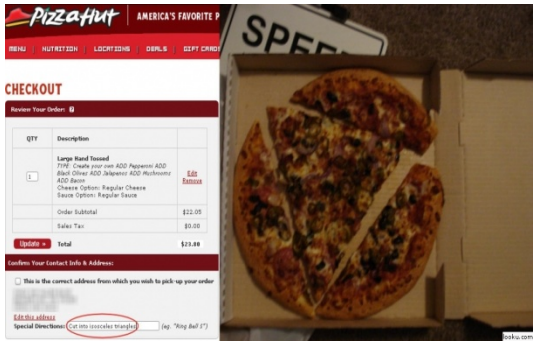
Context is good



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Develop students' proportional reasoning and provide instructional strategies for solving ratio, rate, and proportion problems.

IES – Practice Guide – Fractions, 2010

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Lakers vs Nuggets

- Which player from the Lakers had the best shooting percentage
- Which player from the Lakers had the worst shooting percentage
- Same items for Nuggets
- Which players scored the most points, etc.

5th grade Kara's class

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- I think my first not completely trivial mathematical observation was that the effect on a player's overall batting average from a single day's performance was much greater in the beginning of the season than at the
end.

- It's early I am hitting .200 2 out of 10; I go 3 for 5 and I am now at 5 out of 15 and jump up to .333 an increase of over 100 points to the old BA! But if it's the end of the year and I am still at .200 hitting 100 out of 500 and go 3 for 5, now I am 103 out of 505 and only raise my average to .204. Still a big day, but too many bad days in between!

Ed Dubinsky, who has no idea how much I enjoyed this.

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Favorites

- Write **3** numbers that have some significance to your life – these must be fractions, decimals, percents.
- Exchange lists. Provide random clues for the numbers.
- Guess which numbers fit the clues.

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You can't make this stuff up

- Gettysburg Outlets – July 3, 2009. 50% off sale on all purchases at the Izod store. Sign indicates 50% off the all-store sale.
  - Patron – "well that means it's free."
  - Clerk – "no sir, it's 50% off the 50% off sale."
  - Patron – "well, 50% + 50% is 100% so that means it should be free."
  - This went on for a while. AND, there was a sign indicating 70% off for some items, meaning 70% off the 50% off original sale, which our patron would interpret as the item being free and 20% in cash!

"If you took 10 shots and made 6, tell me how you could determine the percentage of shots made?"

Add, subtract, or multiply. Maybe you can figure it out by multiplying 10 times the 6 of what you missed.

If I average that many points in a game that's how they determine my average and Michael Jordan's average is in the 30's.

60%. Five out of 10 is 50% the next one would be 60%. 5 and 1/2 would be 55% (I don't know the first thing about %, we haven't studied it yet - 5th grader).

"What is percentage?"

Types of scores

Like half of 100 is 50%

Like average, but different.

A percent is per 100 and it's how many out of 100 like 50% is 50 out of 100.

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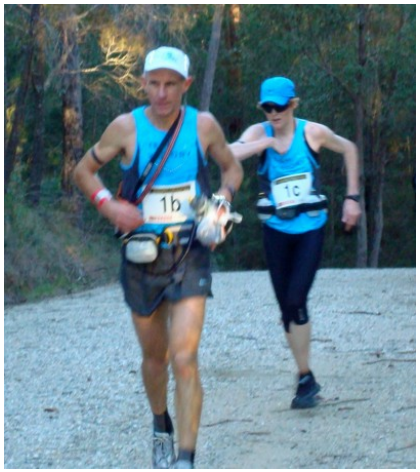
For your next Ultra…

Trail Mix (for 6)

- ½ cup raisins
- ¾ cup peanuts
- 2/3 cup granola
- ½ cup dried fruit
- 2 tablespoons sunflower seeds
- ¼ cup M&Ms



Mix for the whole club – 30 runners
Make that 40 runners…

Adaptation of Bresser, Holtzman, 1999)

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- On a scale 1” = 12 miles. If two places are 4” apart, how far are they away from each other in miles?

1”	12 miles
4”	

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Rational Number Sense

- Using representations of fractions, decimals, and percents interchangeably;
- Comparing and ordering fractions, decimals and percents;
- Using benchmarks to estimate when comparing and ordering and in determining sums, differences, products, and quotients.
- (Adapted from Moss, 2002)

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Concluding Thoughts

- Number sense is elusive, needs nurturing – daily!
- A sense of number breeds confidence.
- Fractions – all of ‘em - are numbers too!
- Decimals and inconsistency…
- Probability – now!
- **Professional development programs should place a high priority on improving teachers’ background in fractions and how to teach them. (**IES – Fractions, 2010**)**

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Look for…

- Empson, Susan and Linda Levi. ***Extending Children's Mathematics: Fractions and Decimals*** (Heinemann, soon).
- McNamara, Julie and Meghan M. Shaughnessy. ***Beyond Pizzas and Pies: 10 Essential Strategies for Supporting Fraction Sense*** (Math Solutions, AT THIS CONFERENCE).
- IES Practice Guide: ***Fractions: Developing Effective Instruction*** (United States Department of Education, online soon).

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http://www.ffennell.com

Want the slides?