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Some things about NCTM you may not know...

NCTM's Curriculum Focal Points:

A Quest for Coherence

Association of Independent Maryland Schools
Baltimore, Maryland

Francis (Skip) Fennell, President National Council of Teachers of Mathematics

&

Professor of Education McDaniel College Westminster, MD November 5, 2007



Coherence



Why?



Why Identify Focal Points?

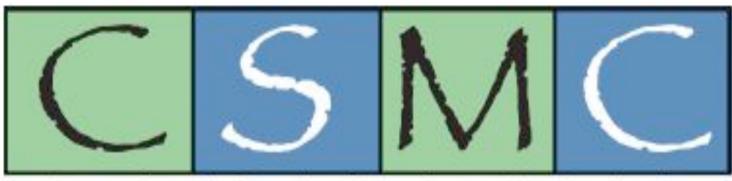
- Address long lists of state learning expectations
- Address "mile wide, inch deep" math curriculum
- Identify the mathematics that should be the focus of instruction and student learning, preK-8
- Begin the discussion of appropriate curricular expectations
- · Identify key mathematical ideas all others build on



Curriculum Focal Points - Timeline

- **Summer 2004**: Park City, UT: ASSM and NCTM Conference to analyze state standards and curriculum convened by Johnny Lott and Kathleen Nishimurra
- April 2005: Release of Standards and Curriculum: A View from the Nation (NCTM).
- March 2005: Austin, TX: NCTM Task Force created by Cathy Seeley, chaired by Barbara Reys
- April 2005: NCTM Board Motion to write Curriculum Focal Points.
- April 2005 Summer 2006: Writing, review, and NCTM Board Approval of the Curriculum Focal Points.
- **September 2006**: Release of the *Curriculum Focal Points PreK-8: A Quest for Coherence* (NCTM).





Center for the Study of Mathematics Curriculum

The Intended Mathematics Curriculum as Represented in State-Level Curriculum Standards: Consensus or Confusion?

WORKING DRAFT (April 14, 2006)

th Number of 4 - Grade Learning

Expectations per State by Content

Strand

	Number & Operation	Geometry	Measurement	Algebra	Data Analysis, Probability & Statistics	Total Number of Learning Expectations
California	16	11	4	7	5	43
Texas	15	7	3	4	3	32
New York	27	8	10	5	6	56
Florida	31	11	17	10	20	89
Ohio	15	8	6	6	13	48
Michigan	37	5	11	0	3	56
New Jersey	21	10	8	6	11	56
North Carolina	14	3	2	3	4	26
Georgia	23	10	5	3	4	45
Virginia	17	8	11	2	3	41

Reys, et al., 2006

What?

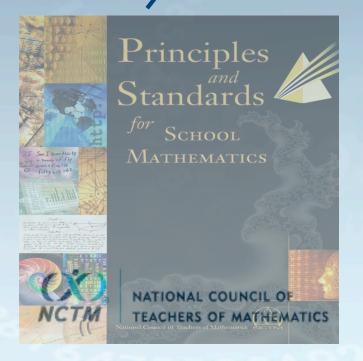


Principles

- Equity
- Curriculum
- Teaching
- Learning
- Assessment
- Technology

Content Standards

- Number/Operations
- Algebra
- Geometry
- Measurement
- Data/Probability



Process Standards

- Problem Solving
- Reasoning
- Communication
- Connections
- Representation

This is NOT the next set of Standards!!!!!

NCTM Curriculum Principle

- A curriculum is more than a collection of activities: it must be
 - -coherent
 - -focused on important mathematics
 - -well articulated across the grades



Principles and Standards for School Mathematics, page 14



NCTM Curriculum Principle

"...a well-articulated curriculum gives teachers guidance regarding important ideas or major themes, which receive special attention at different points in time. It also gives guidance about the depth of study warranted at particular times and when closure is expected for particular times concepts."

Principles and Standards, p. 16



What Are Curriculum Focal Points?

- Important mathematical topics for each grade level, preK-8
- <u>Cohesive clusters</u> of related ideas, concepts, skills, and procedures that form the foundation for higher-level mathematics
- More than a single objective, standard, expectation, or indicator
- Not discrete topics for teachers to present and check off as mastered by students



The Product: Process Standards

Introductory statement for each level, PreK-8:

"It is essential that these focal points be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations."



Curriculum Focal Points and Connections for Grade 2

The set of three curriculum focal points and related connections for mathematics in grade 2 follow. These topics are the recommended content emphases for this grade level. It is essential that these focal points be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

Grade 2 Curriculum Focal Points

Number and Operations: Developing an understanding of the base-ten numeration system and place-value concepts

Children develop an understanding of the base-ten numeration system and place-value concepts (at least to 1000). Their understanding of base-ten numeration includes ideas of counting in units and multiples of hundreds, tens, and ones, as well as a grasp of number relationships, which they demonstrate in a variety of ways, including comparing and ordering numbers. They understand multidigit numbers in terms of place value, recognizing that place-value notation is a shorthand for the sums of multiples of powers of 10 (e.g., 853 as 8 hundreds + 5 tens + 3 ones).

Number and Operations and Algebra: Developing quick recall of addition facts and related subtraction facts and fluency with multidigit addition and subtraction

Children use their understanding of addition to develop quick recall of basic addition facts and related subtraction facts. They solve arithmetic problems by applying their understanding of models of addition and subtraction (such as combining or separating sets or using number lines), relationships and properties of number (such as place value), and properties of addition (commutativity and associativity). Children develop, discuss, and use efficient, accurate, and generalizable methods to add and subtract multidigit whole numbers. They select and apply appropriate methods to estimate sums and differences or calculate them mentally, depending on the context and numbers involved. They develop fluency with efficient procedures, including standard algorithms, for adding and subtracting whole numbers, understand why the procedures work (on the basis of place value and properties of operations), and use them to solve problems.

Measurement: Developing an understanding of linear measurement and facility in measuring lengths

Children develop an understanding of the meaning and processes of measurement, including such underlying concepts as partitioning (the mental activity of slicing the length of an object into equal-sized units) and transitivity (e.g., if object A is longer than object B and object B is longer than object C, then object A is longer than object C). They understand linear measure as an iteration of units and use rulers and other measurement tools with that understanding. They understand the need for equallength units, the use of standard units of measure (centimeter and inch), and the inverse relationship between the size of a unit and the number of units used in a particular measurement (i.e., children recognize that the smaller the unit, the more iterations they need to cover a given length).

Connections to the Focal Points

Number and Operations: Children use place value and properties of operations to create equivalent representations of given numbers (such as 35 represented by 35 ones, 3 tens and 5 ones, or 2 tens and 15 ones) and to write, compare, and order multidigit numbers. They use these ideas to compose and decompose multidigit numbers. Children add and subtract to solve a variety of problems, including applications involving measurement, geometry, and data, as well as nonroutine problems. In preparation for grade 3, they solve problems involving multiplicative situations, developing initial understandings of multiplication as repeated addition.

Geometry and Measurement: Children estimate, measure, and compute lengths as they solve problems involving data, space, and movement through space. By composing and decomposing two-dimensional shapes (intentionally substituting arrangements of smaller shapes for larger shapes or substituting larger shapes for many smaller shapes), they use geometric knowledge and spatial reasoning to develop foundations for understanding area, fractions, and proportions.

Algebra: Children use number patterns to extend their knowledge of properties of numbers and operations. For example, when skip counting, they build foundations for understanding multiples and factors.

Open number lines



Curriculum Focal Points and Connections for Grade 4

The set of three curriculum focal points and related connections for mathematics in grade 4 follow. These topics are the recommended content emphases for this grade level. It is essential that these focal points be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

Grade 4 Curriculum Focal Points

Number and Operations and Algebra: Developing quick recall of multiplication facts and related division facts and fluency with whole number multiplication

Students use understandings of multiplication to develop quick recall of the basic multiplication facts and related division facts. They apply their understanding of models for multiplication (i.e., equal-sized groups, arrays, area models, equal intervals on the number line), place value, and properties of operations (in particular, the distributive property) as they develop, discuss, and use efficient, accurate, and generalizable methods to multiply multidigit whole numbers. They select appropriate methods and apply them accurately to estimate products or calculate them mentally, depending on the context and numbers involved. They develop fluency with efficient procedures, including the standard algorithm, for multiplying whole numbers, understand why the procedures work (on the basis of place value and properties of operations), and use them to solve problems.

Number and Operations: Developing an understanding of decimals, including the connections between fractions and decimals

Students understand decimal notation as an extension of the base-ten system of writing whole numbers that is useful for representing more numbers, including numbers between 0 and 1, between 1 and 2, and so on. Students relate their understanding of fractions to reading and writing decimals that are greater than or less than 1, identifying equivalent decimals, comparing and ordering decimals, and estimating decimal or fractional amounts in problem solving. They connect equivalent fractions and decimals by comparing models to symbols and locating equivalent symbols on the number line.

Measurement: Developing an understanding of area and determining the areas of twodimensional shapes

Students recognize area as an attribute of two-dimensional regions. They learn that they can quantify area by finding the total number of same-sized units of area that cover the shape without gaps or overlaps. They understand that a square that is 1 unit on a side is the standard unit for measuring area. They select appropriate units, strategies (e.g., decomposing shapes), and tools for solving problems that involve estimating or measuring area. Students connect area measure to the area model that they have used to represent multiplication, and they use this connection to justify the formula for the area of a rectangle.

Connections to the Focal Points

Algebra: Students continue identifying, describing, and extending numeric patterns involving all operations and nonnumeric growing or repeating patterns. Through these experiences, they develop an understanding of the use of a rule to describe a sequence of numbers or objects.

Geometry: Students extend their understanding of properties of two-dimensional shapes as they find the areas of polygons. They build on their earlier work with symmetry and congruence in grade 3 to encompass transformations, including those that produce line and rotational symmetry. By using transformations to design and analyze simple tilings and tessellations, students deepen their understanding of two-dimensional space.

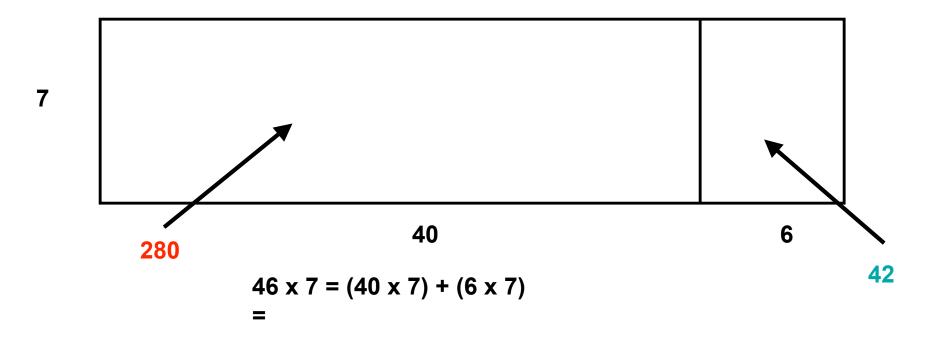
Measurement: As part of understanding twodimensional shapes, students measure and classify angles.

Data Analysis: Students continue to use tools from grade 3, solving problems by making frequency tables, bar graphs, picture graphs, and line plots. They apply their understanding of place value to develop and use stem-and-leaf plots.

Number and Operations: Building on their work in grade 3, students extend their understanding of place value and ways of representing numbers to 100,000 in various contexts. They use estimation in determining the relative sizes of amounts or distances. Students develop understandings of strategies for multidigit division by using models that represent division as the inverse of multiplication, as partitioning, or as successive subtraction. By working with decimals, students extend their ability to recognize equivalent fractions. Students' earlier work in grade 3 with models of fractions and multiplication and division facts supports their understanding of techniques for generating equivalent fractions and simplifying fractions.

Boxes to multiply...

Draw a rectangle to show 46 x 7 = 322



$$280 + 42 = 322$$

• How about 45 x 23

	40	5	
20	40 x 20	5 x 20	45 <u>X 23</u>
3	40 x 3	5 x 3	

Connections – Division & Mental Math

- 275 divided by 5
- Starter problem 250 ÷ 5
- Quinn found 77 beads in a drawer. He was using them to make bookmarks. If he used 5 beads for each bookmark. How many bookmarks could he make?
- Starter problem 50 ÷ 5

Ordering Fractions

Write these fractions in order from least to greatest. Tell how you decided.

• 5/3 5/6 5/5 5/4 5/8

• 7/8 2/8 10/8 3/8 1/8

Draw a picture of it Write with numbers 1/4 or 0.25 Your own connection Use words One fourth; twenty five A quarter is ¼ of a dollar hundredths

Units	Tenths	Hundredths	
	1	3	
1	3		

Curriculum Focal Points and Connections for Grade 7

The set of three curriculum focal points and related connections for mathematics in grade 7 follow. These topics are the recommended content emphases for this grade level. It is essential that these focal points be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

Grade 7 Curriculum Focal Points

Number and Operations and Algebra and Geometry: Developing an understanding of and applying proportionality, including similarity

Students extend their work with ratios to develop an understanding of proportionality that they apply to solve single and multistep problems in numerous contexts. They use ratio and proportionality to solve a wide variety of percent problems, including problems involving discounts, interest, taxes, tips, and percent increase or decrease. They also solve problems about similar objects (including figures) by using scale factors that relate corresponding lengths of the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and identify the unit rate as the slope of the related line. They distinguish proportional relationships (y/x = k, or y = k/x) from other relationships, including inverse proportionality (xy = k, or y = k/x).

Measurement and Geometry and Algebra: Developing an understanding of and using formulas to determine surface areas and volumes of three-dimensional shapes

By decomposing two- and three-dimensional shapes into smaller, component shapes, students find surface areas and develop and justify formulas for the surface areas and volumes of prisms and cylinders. As students decompose prisms and cylinders by slicing them, they develop and understand formulas for their volumes (Volume = Area of base × Height). They apply these formulas in problem solving to determine volumes of prisms and cylinders. Students see that the formula for the area of a circle is plausible by decomposing a circle into a number of wedges and rearranging them into a shape that approximates a parallelogram. They select appropriate two- and three-dimensional shapes to model real-world situations and solve a variety of problems (including multistep problems) involving surface areas, areas and circumferences of circles, and volumes of prisms and cylinders.

Number and Operations and Algebra: Developing an understanding of operations on all rational numbers and solving linear equations

Students extend understandings of addition, subtraction, multiplication, and division, together with their properties, to all rational numbers, including negative integers. By applying properties of arithmetic and considering negative numbers in everyday contexts (e.g., situations of owing money or measuring elevations above and below sea level), students explain why the rules for adding, subtracting, multiplying, and dividing with negative numbers make sense. They use the arithmetic of rational numbers as they formulate and solve linear equations in one variable and use these equations to solve problems. Students make strategic choices of procedures to solve linear equations in one variable and implement them efficiently, understanding that when they use the properties of equality to express an equation in a new way, solutions that they obtain for the new equation also solve the original equation.

Connections to the Focal Points

Measurement and Geometry: Students connect their work on proportionality with their work on area and volume by investigating similar objects. They understand that if a scale factor describes how corresponding lengths in two similar objects are related, then the square of the scale factor describes how corresponding areas are related, and the cube of the scale factor describes how corresponding volumes are related. Students apply their work on proportionality to measurement in different contexts, including converting among different units of measurement to solve problems involving rates such as motion at a constant speed. They also apply proportionality when they work with the circumference, radius, and diameter of a circle; when they find the area of a sector of a circle; and when they make scale drawings.

Number and Operations: In grade 4, students used equivalent fractions to determine the decimal representations of fractions that they could represent with terminating decimals. Students now use division to express any fraction as a decimal, including fractions that they must represent with infinite decimals. They find this method useful when working with proportions, especially those involving percents. Students connect their work with dividing fractions to solving equations of the form ax = b, where a and b are fractions. Students continue to develop their understanding of multiplication and division and the structure of numbers by determining if a counting number greater than 1 is a prime, and if it is not, by factoring it into a product of primes.

Data Analysis: Students use proportions to make estimates relating to a population on the basis of a sample. They apply percentages to make and interpret histograms and circle graphs.

Probability: Students understand that when all outcomes of an experiment are equally likely, the theoretical probability of an event is the fraction of outcomes in which the event occurs. Students use theoretical probability and proportions to make approximate predictions.

Use Percent – Don't Wait!

- Put 2/3; 0.5 and 3/4 in order from smallest to largest.
- It's easy, 0.5 is 50% and 2/3 is 66%, and so it goes first 0.5, then 2/3 and then 3/4 because that's 75%.*

^{*}response by Andy in New Approaches to Teaching the Rational Number System by Joan Moss in How Students Learn: Mathematics in the Classroom, NRC, 2005.

Percent Benchmarks

0%

100% 50% < 10%

About 25% About 75% About 90%

> 50% < 50%

- Lefthanders in the room or class
- Once lived in New Jersey
- Been involved in education > 10 years
- People who were born in Maryland

Missing Numbers

- What's my number?
- 2x + 7 = y
- Rule: Double the number and add 7.
 What's the number if x =

10

100

0.1

0.01

Decimals - What Happens?

Number x 0.05 x 0.48 x 0.9

100
60
24
?

- In general, what happens when you multiply a whole number by: 0.05; 0.48; 0.9?
- Begin thinking of 0.05 as 5% or nickel:dollar, etc.

The Product: Curriculum Focal Points

- Three per grade level, preK-8
- Often represent multiple content strands
- Describe the majority of instruction for a specific grade level
- Taken together across grade levels, provide the major components of a mathematically sound, coherent and cohesive preK-8 curriculum



Connections to the Curriculum Focal Points

- Provide meaningful contexts for the focal points
- Identify connections between strands and across grade levels
- Round out a well-balanced curriculum



Incorporating a Research Base

- Content- and pedagogy-related studies
 (found in publications such as JRME, AERJ, and those from NAEYC)
- National and international measures of students' mathematical proficiencies
 (e.g., NAEP, TIMSS, PISA)
- Schmidt analysis of the Focal Points to A+ countries



How?



Curriculum Focal Points and State and District Leaders

- As a framework for future development of mathematics curriculum
- To identify grade-level targets



Curriculum Focal Points and Teachers

- To design instruction around the question, "What are the most important ideas at my grade level?"
- To provide information about how ideas at one grade level fit with the important ideas in previous and following grades
- To prioritize uses of activities, assessments and other published materials



Curriculum Focal Points and Publishers

As an example for guiding the next generation of instructional materials and related assessments



Curriculum Focal Points and Teacher Educators

To organize preservice and inservice education for developing teachers' knowledge of critical mathematics understandings across the grades



Who did this?



Participation

- Writing group
 - Mathematicians
 - Mathematics educators
 - Teachers
- Outside reviewers
 - Mathematicians and mathematics educators
 - Teachers and supervisors
 - Policymakers



Curriculum Focal Points: What's New

- Priorities focus
- Grade-by-grade descriptions
- Descriptive clusters of content
- More clarification
- Connections



Curriculum Focal Points What's Not New

- Alignment with Principles and Standards for School Mathematics, particularly the Curriculum Principle
- Well-balanced curriculum
- Strong attention to number and operations
- Commitment to problem solving, processes and content
- · Understanding math, doing math, using math



Then What?





September 12, 2006

Arithmetic Problem

New Report Urges Return to Basics in Teaching Math

By JOHN HECHINGER

Critics of 'Fuzzy' Methods Cheer Educators' Findings; Drills Without Calculators

The nation's math teachers, on the front lines of a 17-year Curriculum war, are getting some new marching orders: Make sure students learn the basics.

The New Hork Times nytimes.com

April 13, 2000

Math Teachers Back Return Of Education in Basic Skills

By ANEMONA HARTOCOLLIS

In an important about-face, the nation's most influential group of mathematics teachers announced yesterday that it was recommending, in essence, that the arithmetic be put back into mathematics, urging teachers to emphasize the fundamentals of computation rather than focus on concepts and reasoning.

- Children should master the basic facts of arithmetic that are essential components of fluency with paper-pencil and mental computation and with estimation.
- It is important for children to learn the sequence of steps

 and the reason for them in the paper-and-pencil
 algorithms used widely in our culture.



PreK-4 – Curriculum and Evaluation Standards, NCTM, 1989, p.47



- Knowing basic number combinations the single digit addition and multiplication pairs and their counterparts for subtraction and division – is essential.
- Equally essential is computational fluency having and using efficient and accurate methods for computing. Fluency may be manifested in using a combination of mental strategies and jottings on paper or using an algorithm with paper and pencil, particularly when the numbers are large, to produce accurate results quickly. Regardless of the particular algorithm used, students should be able to explain their method, understand that many methods exist, and see the usefulness of methods that are efficient, accurate, and general.

Number & Operations, Principles and Standards for School Mathematics, NCTM, 2000, p. 32



And then....

Over 825,000 downloads later

Most popular NCTM publication – sales



NCTM has met with over 20 states

Use of the Curriculum Focal Points to assist in revising their state standards

Survey – Curriculum Center Project



Selected National Presentations

- National Math Panel Presentation September, 2006
- Capitol Senate and House Aides October, 2006
- AMS, MAA October, 2006 and January, 2007
- Major Publishers November, 2006
- CBMS Presentation December, 2006
- Department of Education MSP Meetings
- National Title I meeting January, 2007
- AMTE January, 2007
- Curriculum Center Conference, Washington, D.C. February, 2007
- Lieutenant Governor's Retreat March, 2007
- AERA April, 2007
- Many others...



Concerns

- Confusion Understanding what is meant by a focal point – target, area of emphasis, a focus.
- Confusion Concept vs Content
- Will states and school districts "drop" topics?
- When will the state tests change?
- Push back within NCTM
 - Why this topic at this grade level (see above)?
 - Should NCTM have taken more time?
 - What about high school?



And...





Sunday, Nov. 19, 2006 **How to End the Math Wars We have a new formula for teaching kids. Don't let ideology ruin it this time**By CLAUDIA WALLIS

Local Schools to Study Whether Math - Topics = Better Instruction

The Washington Post

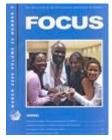
By Daniel de Vise Washington Post Staff Writer December 5, 2006



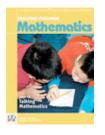
Focal Points for Pre-K8 math May 2007



Mathematics Teaching in the Middle School – Department 2007-08



MAA's Focus April 2007



Teaching Children
Mathematics –
Department 2008

Questions...

- Can curriculum/standards designed around a few key ideas structure a comprehensive program?
- Can assessments focus on priorities and problem solving?
- How might textbooks/materials look different if structured around focal points?
- How can state/federal policies best support rich, deep appropriate mathematics for every student?



The Goal: Curriculum Focal Points and Improved Mathematics Education

- Guidance for schools and states in the design of curricula and assessment that target the most important topics
- Focus for teachers that gives sufficient time for students to understand concepts and develop and apply skills necessary for future mathematics
- Clear direction for students and parents on the importance of deep understanding of particular topics at each grade level



Next Steps

- Essentials
- Focus On...: grade level and grade bands
- High School Curriculum Project
- The NMP and the Focal Points...
- The NSB and the Focal Points





- ACT study notes gap between U.S. high school curriculum and college expectations.
 - Colleges generally want all incoming students to attain an in-depth understanding of a selected number of fundamental skills and concepts in their high school courses, while high schools tend to provide less indepth instruction of a broader range of skills and topics.

April, 2007

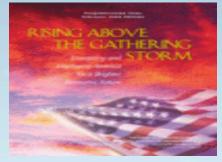


Competitiveness



86% of U.S. voters believe that the U.S.
must increase the number of workers with
a background in science and mathematics
or America's ability to compete in the
global economy will be diminished.

- The Tom Friedman effect!
- Competitiveness an advantage for new ideas



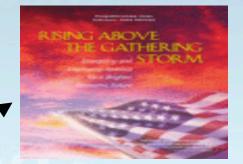


From the Secretary...

 "We must encourage students to take more advanced math and science classes. Employers today need workers with 'pocket protector' skills – creative problem solvers with strong math and science backgrounds."

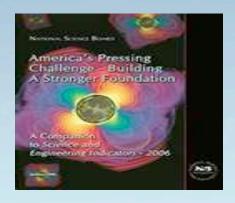
Margaret Spellings, June 21, 2007





America's Pressing Challenge: Building A Stronger Foundation

 Our nation must devote the necessary resources now to revitalize our precollege STEM education system. We cannot wait for a new *Sputnik* episode to energize our population to rise to this challenge – we must recognize the existing crisis and take the necessary actions.



NSF, 2006



 ...math and science are the keys to innovation and power in today's world, and American parents had better understand that the people who are eating their kids' lunch in math are not resting on their laurels.



Opportunities

- The Immediate Horizon
 - National Mathematics Panel
 - Math Now
 - NSB Initiative



Algebra Issues

- What is algebra?
- When?
- Who's the teacher?
- And, then what? What math? When?
- Stay tuned for the NMP Report



If we don't step up to the challenge of finding and supporting the best teachers we'll undermine everything else we are trying to do to improve our schools.



"I didn't call myself anything. I was more than a teacher. And less. In the high school classroom you are a drill sergeant, a rabbi, a shoulder to cry on, a disciplinarian, a singer, a scholar, a clerk, a referee, a clown, a counselor, a dress-code enforcer, a conductor, an apologist, a philosopher, a collaborator, a tap dancer, a politician, a therapist, a fool, a traffic cop, a priest, a mother-father-sister-brother-aunt-uncle, a bookkeeper, a critic, a psychologist, the last straw."

Your Questions?





NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

www.nctm.org

www.ffennell.com