

Number Sense is a Critical Foundation:

What does this mean, and how do we get there?

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&

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*NCTM Regional Conference
Oklahoma City, OK*

1	2	3	4
11	12	13	14
21	22	23	24
31	32	33	34

All students should leave elementary school with a strong sense of number

What does that mean?

How do we do that?

Principles and Standards for School Mathematics

Content Standards

- **Number and Operations**
- Algebra
- Geometry
- Measurement
- Data Analysis and Probability



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Some History!

Number Sense

- Number Meaning
- Relationships
- Magnitude
- Operation Sense
- Real Life Number Sense - Applications

Howden, 1989



- In its most fundamental form, number sense entails an ability to immediately identify the numerical value associated with small quantities;
- ...this more highly developed form of number sense should extend to numbers written in fraction, decimal, and exponential forms.
- ...poor number sense interferes with learning algorithms and number facts and prevents use of strategies to verify if solutions to problems are reasonable.



Do you have a sense of number?

- Is 4×12 closer to 40 or 50?
- How many paper clips can you hold in your hand?
- If the restaurant bill was \$119.23, how much of a tip should you leave?
- How long will it take to make the 50 mile drive to Washington, D.C.?
- If a 10-year old is 5' tall, how tall will the child be at age 20?



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Policy and Political Issues

- Number sense is developed!
- Number sense includes automaticity!
- Where does this fit in any state's curricular standards?
- This is more than whole numbers!



Number Meaning - Critical Issues

Whole Numbers:

- Number Meaning
- Counting - Counting on, Counting back
- Composing and Decomposing

Composing and Decomposing
Number is Critical!

Math Wall Activities

24

73

49

Today's Date

Important Benchmarks

- Early
 - Ten
 - Hundred
- Later On
 - Thousand
 - Million



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0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

100 Chart Patterns

- Numbers that have a difference of 1
- Numbers that have a 4 in them
- Every other number
- Even numbers
- Prime numbers
- Multiples of 5, 6, 3
- Divisible by 4
- Many, many more

100 Chart Puzzles

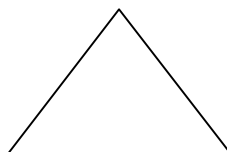
	15			
24		26		

100

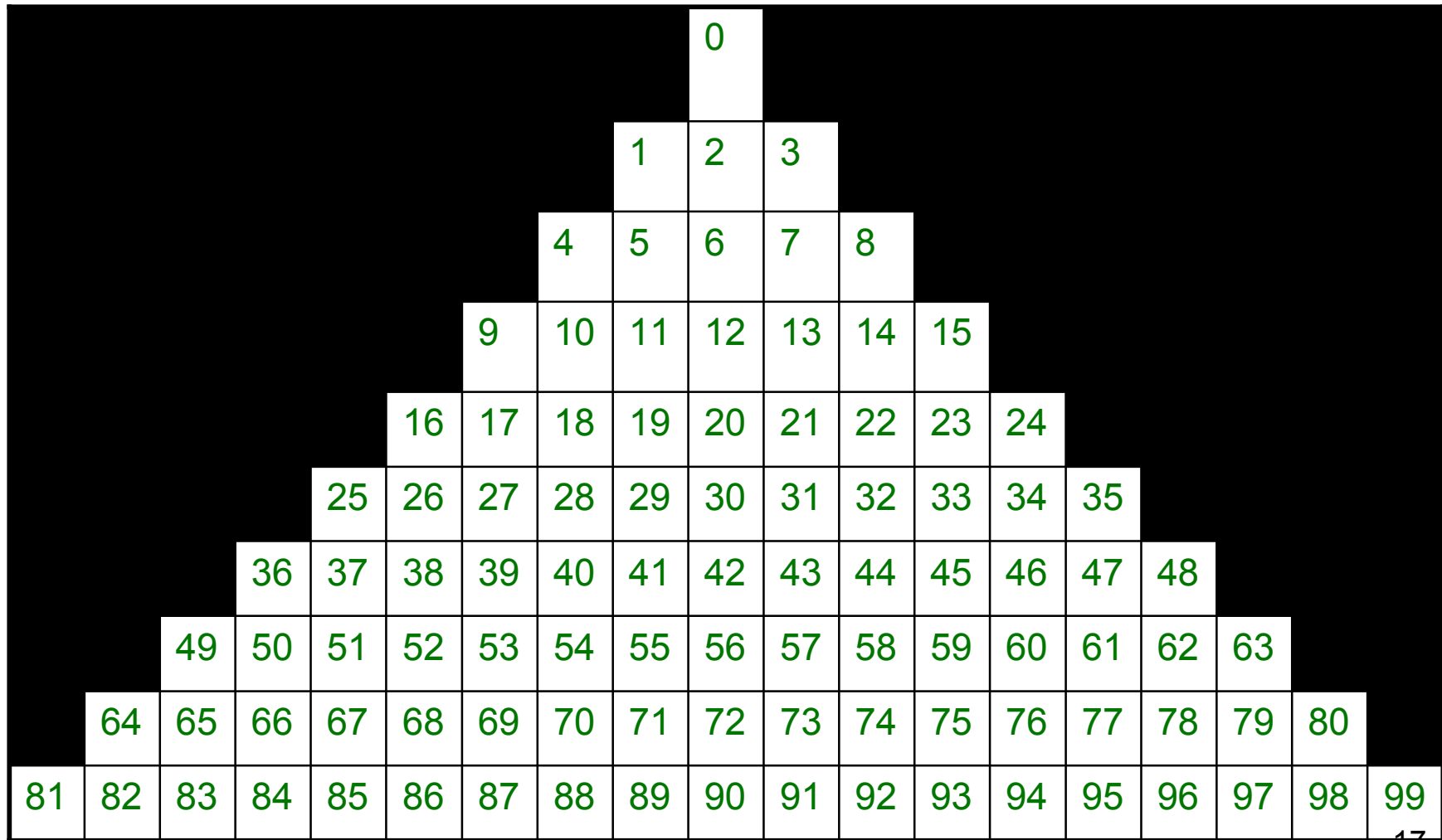
- 100 is a big number when it's:
- 100 is a small number when it's:

100 Chart Equations

- Circle 38. Add 10. Add 1. Subtract 9. Add 5. New number is _____.
- Circle 6. Add 30. Subtract 8. New number is _____.
- $45 - 10 + 7 =$
- Write your own:



And how about this chart?





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Sun	Mon	Tues	Wed	Thur	Fri	Sat
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

My number of the day*

- The number before my number is _____
- The number after my number is _____
- _____ is 10 more than my number.
- _____ is 50 more than my number.
- _____ is 100 more than my number.
- You can find my number by counting by _____'s.

***children select a special number each day**

My number of the day*

- Multiply your number by 4: _____
- Subtract 1: _____
- What is the new number? _____
- How is the new number different from your number of the day?
- $4x - 1 = n$



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Name something that helps you attach meaning to each number below:

- 25
- 50
- 500
- 75
- 60
- 36
- 30

Favorites

- Write **3** numbers that have some significance to your life.
- Exchange lists. Provide random clues for the numbers.
- Guess which numbers fit the clues.



What's my number?

- Start with n . Double it. Now it's?
 - What is $n \times 4$?
 - What is $n \times 10$?
 - What is $n \times 100$?
-
- What is $\frac{1}{2} n$? What is 50% of n ?
 - What is $\frac{1}{4} n$? What is 25% of n ?
 - Name two numbers n falls between.



Today's Secret Number (Mr. x)

- It is less than 3×8
- It is an even number
- It is more than 2 weeks
- It is not a multiple of 8
- It is divisible by 10
- What is today's number? _____

Today's Target is 36

- Try to make today's target by:
 - Adding 2 numbers
 - Finding the difference of 2 numbers
 - Multiplying 2 numbers
 - Adding 3 numbers
 - Multiplying 3 numbers
 - Multiplying and subtracting
 - YOUR own method!

McIntosh, Reys, Reys, and Hope (1997)

Number Sense Language

- bunch
- pile
- flock
- herd
- stack
- handful
- basket
- cord
- crowd



Basic Facts

- Commutative Property
- Multiplying by 0
- Multiplying by 1
- Squares
- Doubles - 2's facts
- Nickels Facts - 5's facts



$$9 \times 0 = 0$$

$$9 \times 1 = 9$$

$$9 \times 2 = 18$$

$$9 \times 3 = 27$$

$$9 \times 4 = 36$$

$$9 \times 5 = 45$$

$$9 \times 6 = 54$$

$$9 \times 7 = 63$$

$$9 \times 8 = 72$$

$$9 \times 9 = 81$$

$$4 \times 0 = 0$$

$$4 \times 1 = 4$$

$$4 \times 2 = 8$$

$$4 \times 3 = 12$$

$$4 \times 4 = 16$$

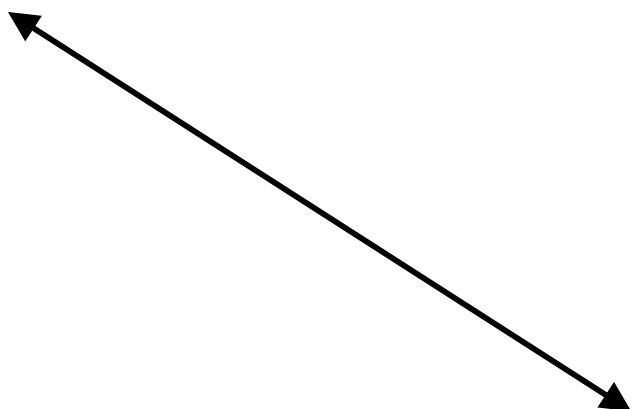
$$4 \times 5 = 20$$

$$4 \times 6 = 24$$

$$4 \times 7 = 28$$

$$4 \times 8 = 32$$

$$4 \times 9 = 36$$





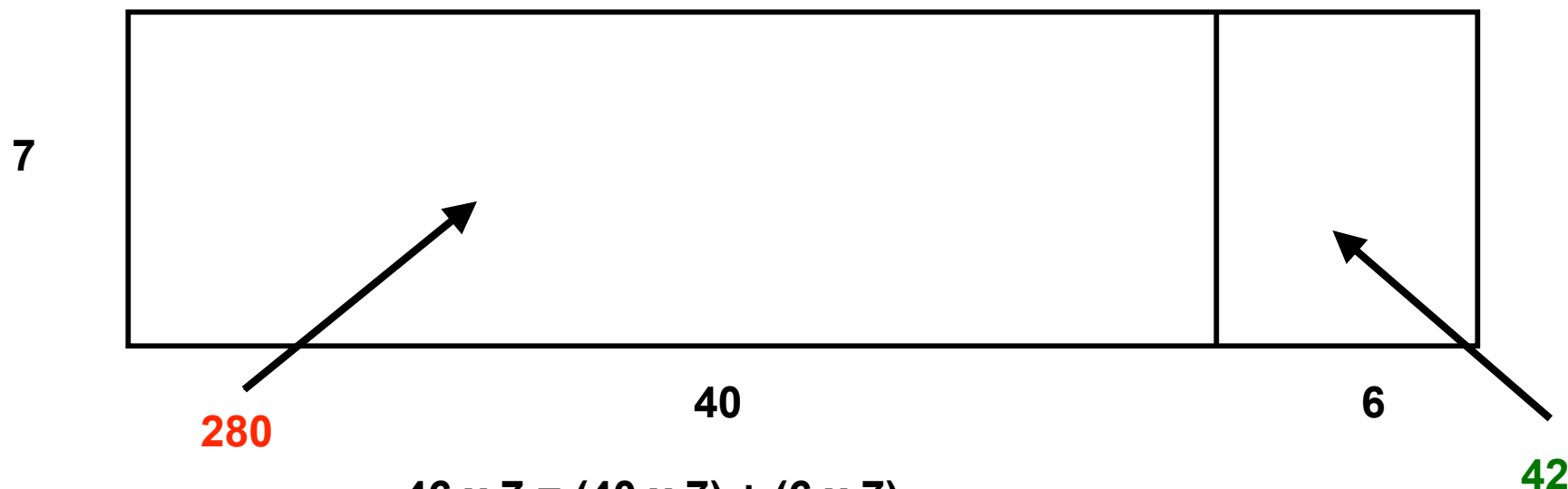
- Finding and using patterns and other thinking strategies greatly simplifies the task of learning multiplication tables.

Thornton, 1978

- Children need to identify individual products rapidly. **Little is known** about how children acquire this fluency or what experiences might be of most help.

Boxes to multiply...

- Draw a rectangle to show $46 \times 7 = 322$



$$46 \times 7 = (40 \times 7) + (6 \times 7)$$
$$=$$

$$280 + 42 = 322$$

- How about 45×23

	40	5	
20	40 x 20	5 x 20	45 X 23
3	40 x 3	5 x 3	

True or False - 818

- Number of students in your school?
- Number of people in your town?
- Number of players on the team?
- Number of pennies in a collection?
- Closer to 500 or 1,000?
- > 500
- > 750

Estimate or Exact?

- Your school bus number.
 - When to leave for school in the AM.
 - When a flight will leave the airport.
 - Total bill at a restaurant.
-
- When do you estimate?
 - When must you have an exact response?



How many digits in the answer?

- $174 + 689 =$
- $134 + 989 =$
- $1,246 - 348 =$
- $874 - 567 =$
- $12 \times 48 =$
- $12 \times 336 =$
- $2,344 \div 4 =$



Estimation – Some Thoughts

- Estimating Magnitude – should begin early and occur often.
- Children are initially uncomfortable with computational estimation.
- The language of computational estimation is adult language. Children seem OK with such language as they grow – experientially.

Think about this - A test!

- Four 2-digit numbers were added together.
 - The sum is 100
 - One of the addends is in the 20's.
 - One of the addends is in the 50's.
 - What can you say about the other two addends?

Between - Density

- Name a number between 17 and 25.
- Name a number between 76 and 77.
- Name a number between 3.49 and 3.53.
- Name a number between 3.4 and 3.5.
- Name a number between $\frac{1}{8}$ and $\frac{1}{4}$.



Estimation

- How many 1-digit numbers are there? 2-digit numbers? 3-digit numbers?
- The toll road is 243 miles long. If you traveled at a speed of 61 mph, about how many hours will you be on the toll road?
- The height of full grown human is about 21 times the length of the middle finger.

Plastics...

A major goal for K-8 mathematics
should be proficiency with
fractions...



- Fractions are a major area of study in upper elementary school mathematics. It is time to shift the emphasis and redefine the goal of fraction instruction from learning computation rules to developing fraction operation sense (Huinker, 2002).
- Do we do this?

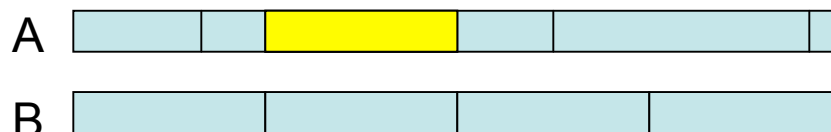
NMAP - Student Preparation

- The first question concerned the adequacy of student preparation coming into the Algebra I classes. The topics that were rated as especially problematic were:
 - Rational numbers;
 - Solving word problems, and;
 - Basic study skills.

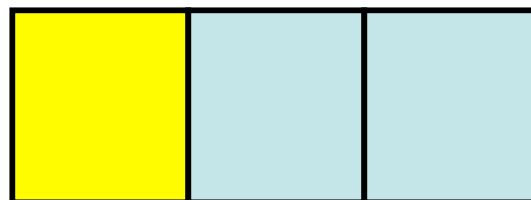
Final Report on the National Survey of Algebra Teachers for the National Math Panel, NORC, September, 2007

Fraction beginnings...

- fractions as relational quantities
 - the yellow piece is what fraction of A?
of B?



- which one is larger, $\frac{1}{2}$ or $\frac{1}{3}$?



Fraction Sorting

- Sort the fractions below as near: 0, $\frac{1}{2}$, or 1

$$\frac{4}{7}$$

$$\frac{1}{7}$$

$$\frac{8}{9}$$

$$\frac{3}{5}$$

$$\frac{2}{3}$$

$$\frac{1}{10}$$

$$\frac{4}{8}$$

$$\frac{6}{11}$$

$$\frac{4}{5}$$

$$\frac{2}{12}$$

$$\frac{9}{12}$$

$$\frac{5}{12}$$

$$\frac{1}{8}$$

$$\frac{3}{8}$$

$$\frac{4}{9}$$

$$\frac{7}{14}$$

- What's alike about all fractions near 1? Near 0?

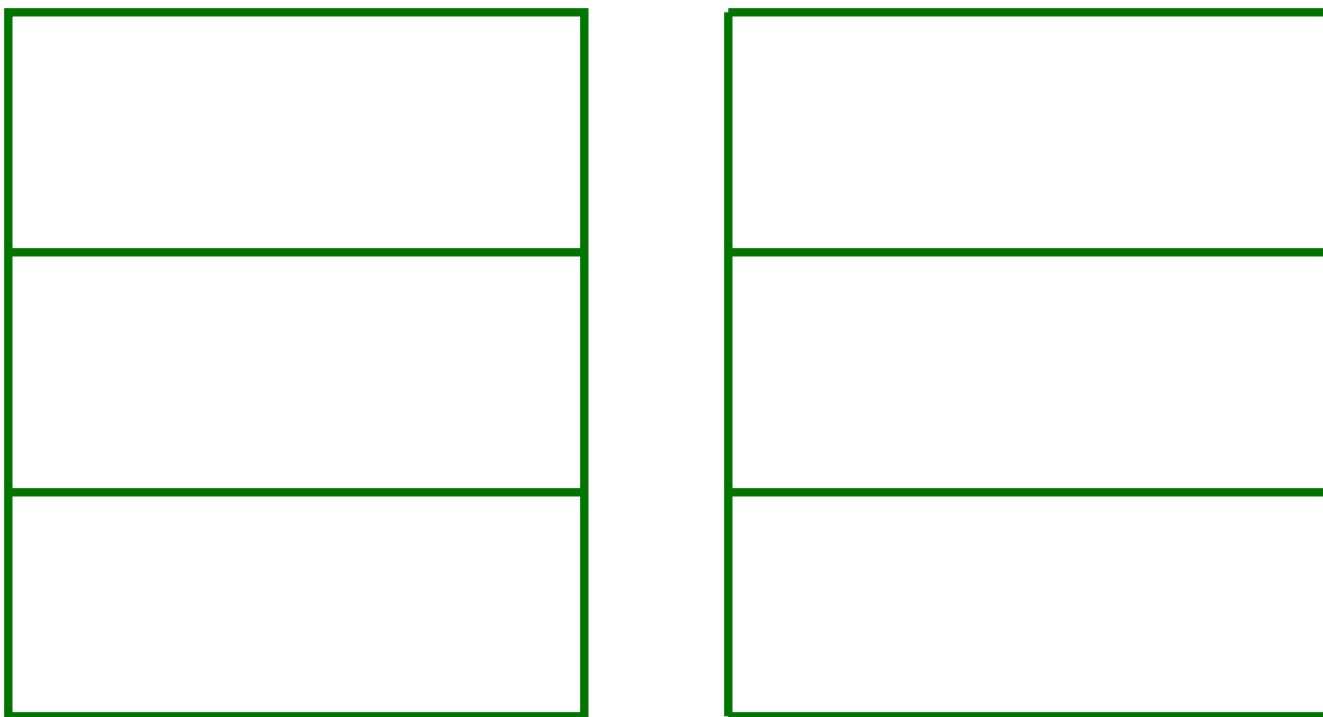
Partitioning

- How can we share eleven hoagies (aka subs) among four people?
- How can we share eleven hoagies (aka subs) among five people?





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Could the drawing above be shaded to show $\frac{5}{3}$?
 $\frac{5}{6}$? Why is this a concern?



Now what?

- There are 25 students in our class. Each student will get $\frac{1}{4}$ of a pizza. Your job is to find out how many pizzas we should order. Be sure to show your work.
- How many pizzas should we order?

Fractions!



$$\frac{3}{4}$$

- What happens to the value of the fraction if the numerator is increased by 1?
- What happens to the value of the fraction if the denominator is decreased by 1?
- What happens to the value of the fraction if the denominator is increased?



$$\frac{3}{4}$$

- What happens to the value of the fraction if the numerator AND denominator are doubled?

Ordering Fractions

Write these fractions in order from least to greatest. Tell how you decided.

- $\frac{5}{3}$ $\frac{5}{6}$ $\frac{5}{5}$ $\frac{5}{4}$ $\frac{5}{8}$
- $\frac{7}{8}$ $\frac{2}{8}$ $\frac{10}{8}$ $\frac{3}{8}$ $\frac{1}{8}$

- Tell me about where $\frac{2}{3} + \frac{1}{6}$ would be on this number line (Cramer, Henry, 2002).



Sense Making:

“ $\frac{2}{3}$ is almost 1, $\frac{1}{6}$ is a bit more, but the sum is < 1 ”



- List three fractions between $\frac{1}{4}$ and $\frac{1}{2}$
 - ?
 - ?
 - ?

Decimals - What Happens?

Number	$\times 0.05$	$\times 0.48$	$\times 0.9$
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100

60

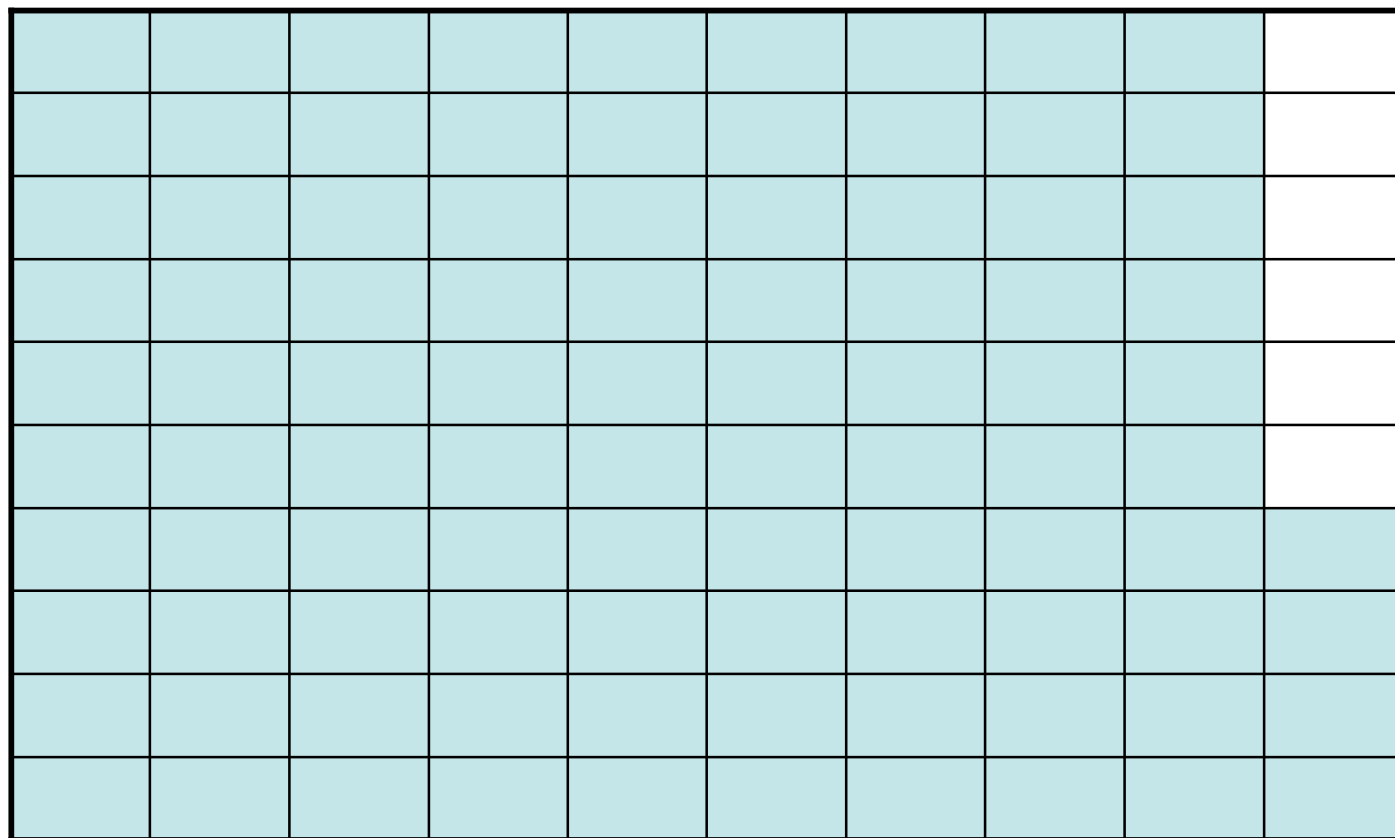
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?

- In general, what happens when you multiply a whole number by: 0.05; 0.48; 0.9?
- Begin thinking of 0.05 as 5% or nickel:dollar, etc.



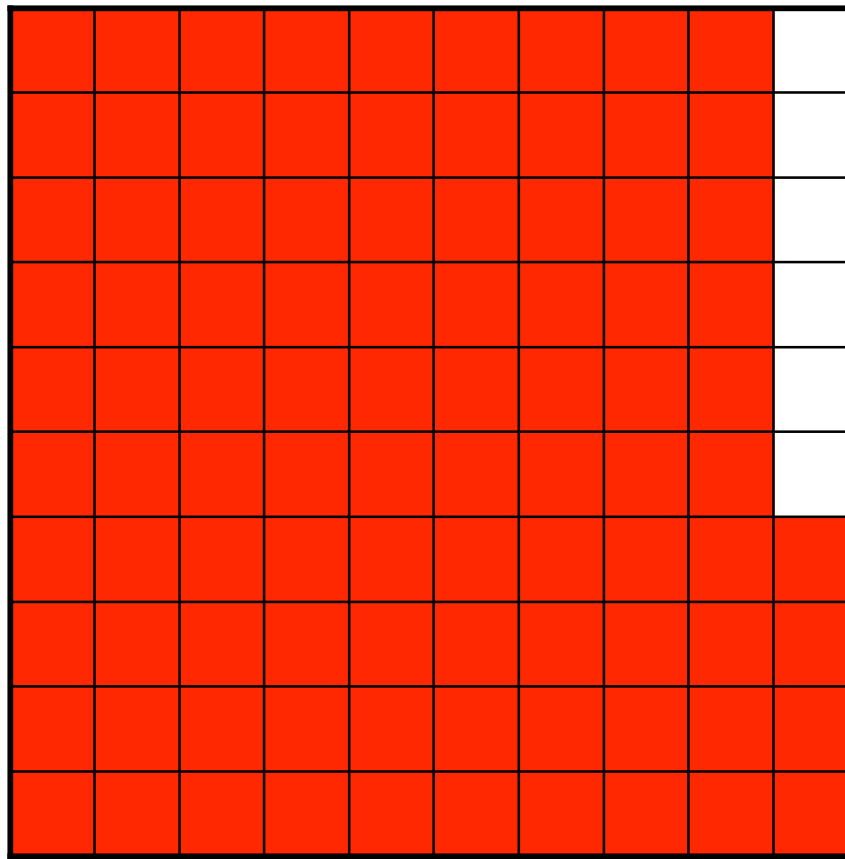
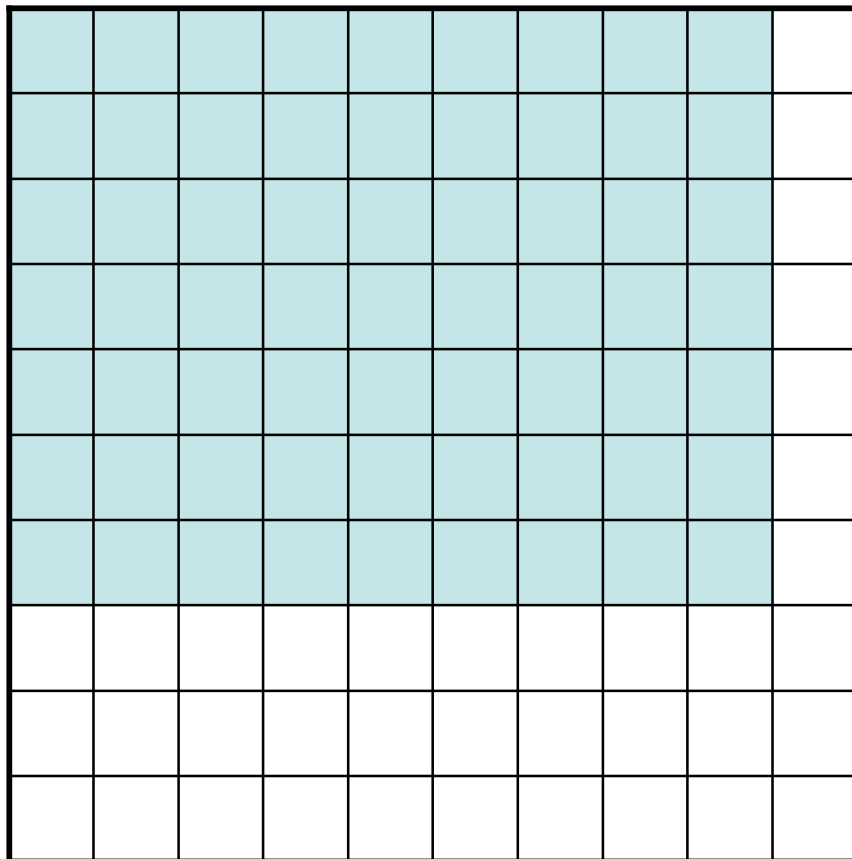
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Close to 1? How much?



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How much?



What Happens Here?

• $1/2 \times 3/4$	$< \text{ or } >$	$3/4$
--------------------	-------------------	-------

• $3/4 \times 1/2$	$< \text{ or } >$	$1/2$
--------------------	-------------------	-------

• $1/2 \div 3/4$	$< \text{ or } >$	$1/2$
------------------	-------------------	-------

• $3/4 \div 1/2$	$< \text{ or } >$	$3/4$
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Switching Numerators and Denominators

- $8/9 \times 3/4 =$
 - $3/9 \times 8/4 =$
 - $1/3 \times 2 = 2/3$
-

- $3/16 \times 4/9 =$
- $4/16 \times 3/9 =$
- $1/4 \times 1/3 = 1/12$

Try one yourself!
Switch the
denominators!

Use Percent – Don't Wait!

- Put $\frac{2}{3}$; 0.5 and $\frac{3}{4}$ in order from smallest to largest.
- It's easy, 0.5 is 50% and $\frac{2}{3}$ is 66%, and so it goes first 0.5, then $\frac{2}{3}$ and then $\frac{3}{4}$ because that's 75%.*

*response by Andy in New Approaches to Teaching the Rational Number System by Joan Moss in How Students Learn: Mathematics in the Classroom, NRC, 2005.

Percent Benchmarks

	0%	
100%	50%	< 10%
~25%	~75%	~90%
	> 50%	< 50%

- Lefthanders in the room or class
- Once lived in **New Jersey**
- Been involved in education > 10 years
- People who were born in **Oklahoma**

Missing Numbers

- What's my number?
- $2x + 7 = y$
- Rule: Double the number and add 7.
What's the number if $x =$

10

100

0.1

0.01

Where's the decimal point go?

- $8.432 \times 5.75 = 48484$
- $3.044 \times 16.5 = 50226$
- $3.326 \times 0.32 \times 31.5 = 3352608$
- $306.15 \div 75.4 = 40603448$

Name that decimal!

- A decimal > 3 and < 4
- A decimal > 2.15
- A decimal < 3
- Two decimals whose sum = 1
- Three decimals whose sum < 0.8
- Four decimals whose sum = 2.35



"Say something about the Order of Operations"

Like numerical order and you do it in a certain order.

Way to do things like multiplying problems; different signs are presented and the order you need to do them in.

First make sure they are clean, then take their clothes off, then cut, then sew up, then back to their room.

Time and Fractions

$$1/2 + 1/4 = 3/4; 6 + 3 = 9 \text{ of } 12 \text{ or } 3/4$$

$$5/6 - 1/2 = 1/3; 10 - 6 = 4 \text{ of } 12 \text{ or } 1/3$$

$$1/4 + 2/3 = 11/12$$

$$2 \frac{1}{4} \div \frac{3}{4} =$$

$$1/3 \times 7 = 2 \frac{1}{3}$$



Rational Number Sense

- Using representations of fractions, decimals, and percents interchangeably;
- Comparing and ordering fractions, decimals and percents;
- Using benchmarks to estimate when comparing and ordering and in determining sums, differences, products, and quotients.
- (Adapted from Moss, 2002)

Concluding Thoughts

- Number sense is elusive
- Number sense should be nurtured – every day!
- A sense of number breeds confidence.
- Number sense is not the final chapter in a 12 chapter book!
- Fractions – all of ‘em - are numbers too!



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Now, how do we get there?

- Time
- Importance of number sense – all numbers
- Development of concepts, proficiency, and problem solving

Want the slides???

<http://www.ffennell.com>