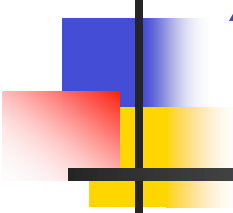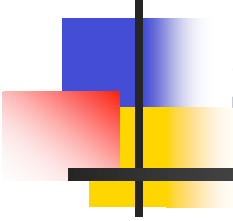# Number Sense:
A Critical Foundation for Higher-Level Mathematics

Francis (Skip) Fennell, President
National Council of Teachers of Mathematics
&
Professor of Education
McDaniel College

Thinking Mathematically 2007
Bucknell University
October 15, 2007

All students should leave elementary school with a strong sense of number

What does that mean?

How do we do that?

Principles and Standards for School Mathematics

Content Standards

- ## Number and Operations
- ## Algebra
- ## Geometry
- ## Measurement
- ## Data Analysis and Probability

Some History!

Number Sense

- Number Meaning
- Relationships
- Magnitude
- Operation Sense
- Real Life Number Sense - Applications

Howden, 1989

Do you have a sense of number?

- Is 4 x 12 closer to 40 or 50?
- How many paper clips can you hold in your hand?
- If the restaurant bill was $119.23, how much of a tip should you leave?
- How long will it take to make the 50 mile drive to Washington, D.C.?
- If a 10-year old is 5' tall, how tall will the child be at age 20?

Policy and Political Issues

- Number sense includes automaticity!
- Number sense is developed!
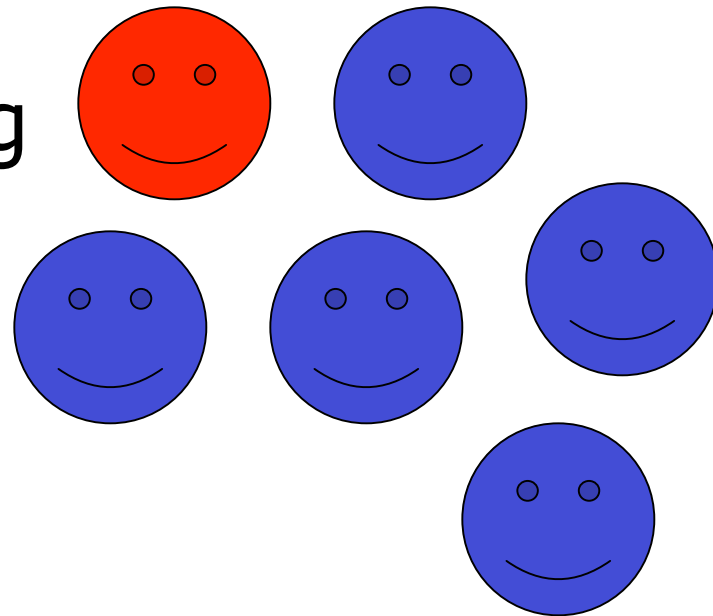- Where does this fit in any state's curricular standards?

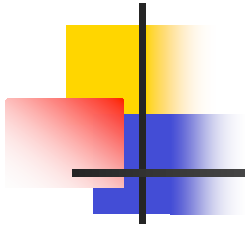# Number Meaning - Critical Issues

- Number Meaning
- Counting - Counting on, Counting back
- Composing and Decomposing

It starts with counting!
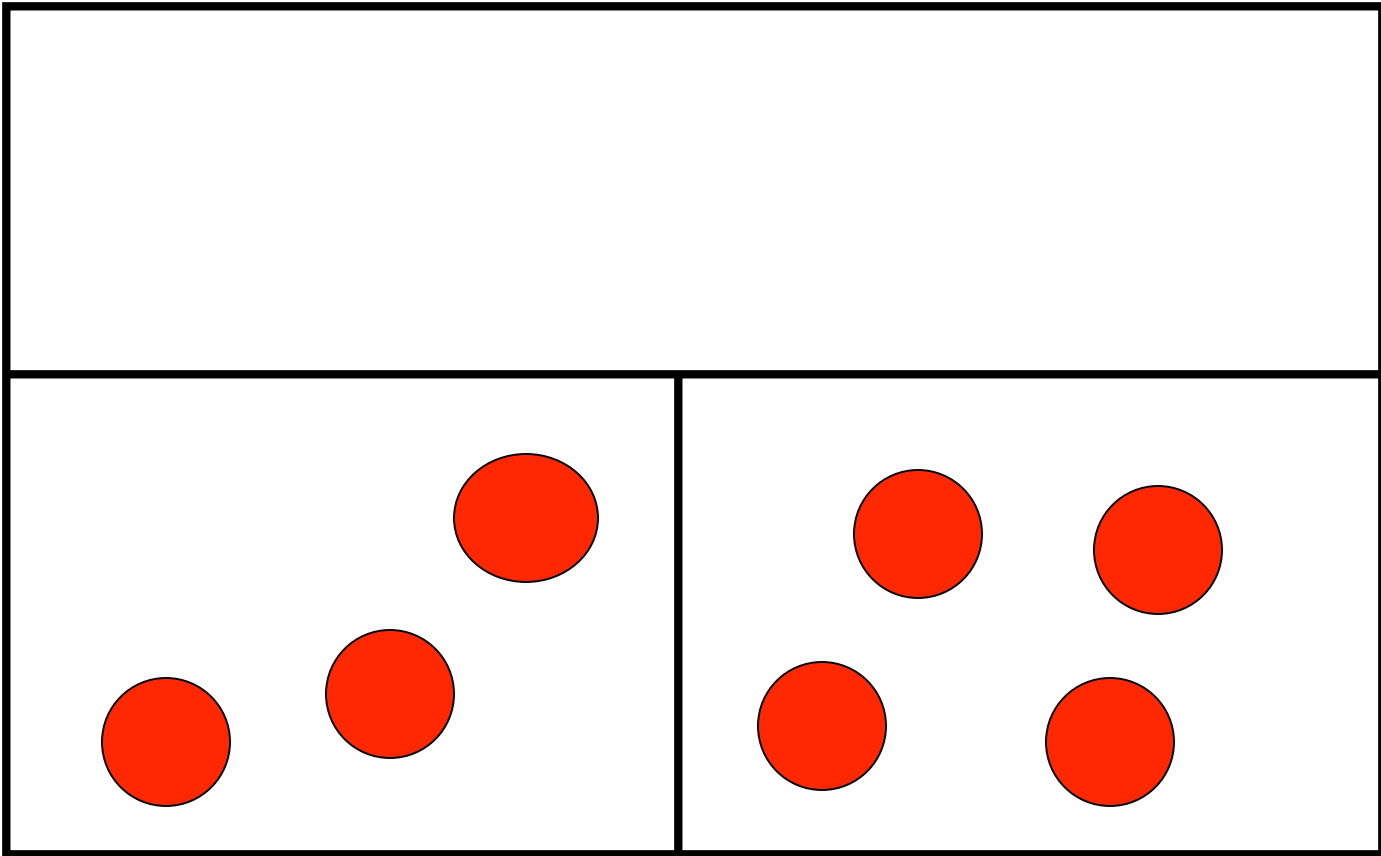
- Oral Counting
- Rational Counting
- Subitizing

- *Counting On*
- *Counting Back*
- *Skip Counting*

- For students in grades K-2, learning to see the part-part-whole relations in addition and subtraction is one of their most important accomplishments in arithmetic.

Resnick, L.B. (1983)

Important Benchmarks

- **Early**
  - Ten
  - Hundred

- **Later On**
  - Thousand
  - Million

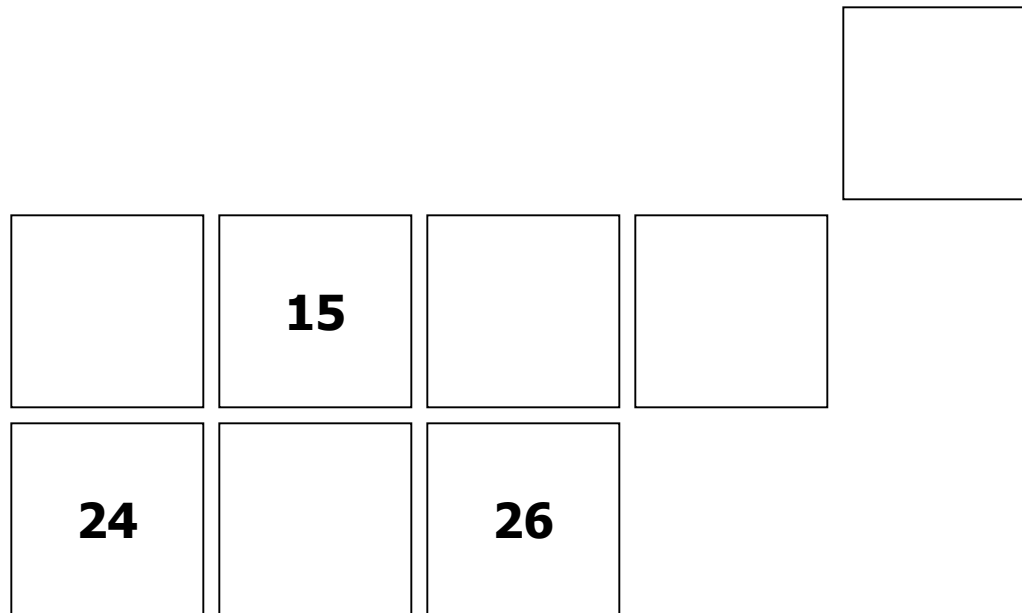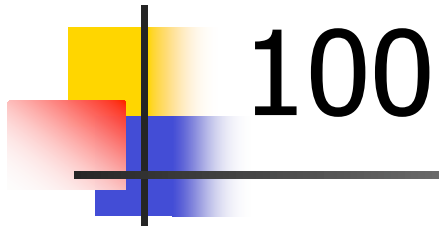| 0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

100 Chart Patterns

- Numbers that have a difference of 1
- Numbers that have a 4 in them
- Every other number
- Even numbers
- Prime numbers
- Multiples of 5, 6, 3
- Divisible by 4
- Many, many more

100 Chart Puzzles

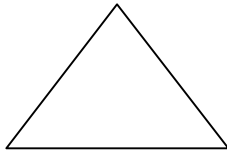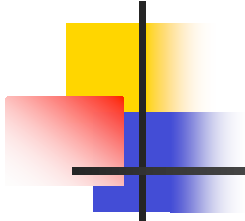|			
	15		
24		26	

100

- 100 is a big number when it's:

- 100 is a small number when it's:

100 Chart Equations

- Circle 38. Add 10. Add 1. Subtract 9. Add 5. New number is _____.

- Circle 6. Add 30. Subtract 8. New number is _____.

- $45 - 10 + 7 =$ △

- Write your own:

Sun	Mon	Tues	Wed	Thur	Fri	Sat
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

Thinking about 1,000,000

- Make tallies for one minute. How many did you make?

- How long would it take to make 1,000 tallies?

- How long would it take to make 1,000,000 tallies?

Math Wall Activities

24

73

49

Today's Date

What's Next? Why?

- 5, 15, 20, 30, 35, 45,…

- 1, 1, 2, 4, 3, 9,…

- Friday, Thursday, Thursday, Friday, …

- _____, 25, _____

My number of the day*

- The number before my number is _____
- The number after my number is _____
- _____ is 10 more than my number.
- _____ is 50 more than my number.
- _____ is 100 more than my number.
- You can find my number by counting by _____'s.

  *children select a special number each day

My number of the day*

- Multiply your number by 4: _____
- Subtract 1: _____
- What is the new number? _____

- How is the new number different from your number of the day?
- $4x - 1 = n$

Algebra - Equations

Name something that helps you attach meaning to each number below:

- 25
- 50
- 500
- 75
- 60
- 36
- 30

Favorites

- Write **3** numbers that have some significance to your life.

- Exchange lists. Provide random clues for the numbers.

- Guess which numbers fit the clues.

What's my number?

- Start with n. Double it. Now it's?
- What is n x 4?
- What is n x 10?
- What is n x 100?
- What is ½ n? What is 50% of n?
- What is ¼ n? What is 25% of n?
- Name two numbers n falls between.

Today's Secret Number (Mr. x)

- It is less than 3 x 8
- It is an even number
- It is more than 2 weeks
- It is not a multiple of 8
- It is divisible by 10
- What is today's number? _____

True or False - 818

- Number of students in your school?
- Number of people in your town?
- Number of players on the team?
- Number of pennies in a collection?
- Closer to 500 or 1,000?
- > 500
- > 750

Division and Fair Shares

- How would you share 11 subs among 4 people?

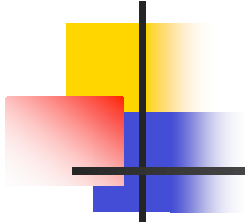- How would you share 11 subs among 5 people?

Number Sense Language

- bunch
- pile
- flock
- herd
- stack
- handful
- basket
- cord
- crowd

Basic Facts

- Commutative Property
- Multiplying by 0
- Multiplying by 1
- Squares
- Doubles - 2's facts
- Nickels Facts - 5's facts

9 x 0 = 0	4 x 0 = 0
9 x 1 = 9	4 x 1 = 4
9 x 2 = 18	4 x 2 = 8
9 x 3 = 27	4 x 3 = 12
9 x 4 = 36	4 x 4 = 16
9 x 5 = 45	4 x 5 = 20
9 x 6 = 54	4 x 6 = 24
9 x 7 = 63	4 x 7 = 28
9 x 8 = 72	4 x 8 = 32
9 x 9 = 81	**4 x 9 = 36**
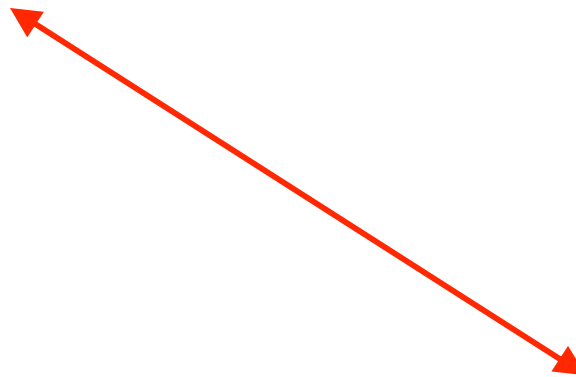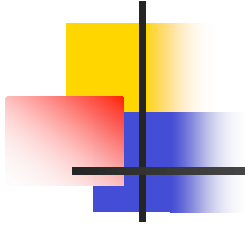
- Finding and using patterns and other thinking strategies greatly simplifies the task of learning multiplication tables.

  Thornton, 1978

- Children need to identify individual products rapidly. **Little is known** about how children acquire this fluency or what experiences might be of most help.

Mystery Facts

- The sum of the digits in this 2-digit number is 9. The difference between the digits is 3. A number that fits this description is _____. Multiplication fact(s):

- The tens digit in this 2-digit number is one-fourth of the ones digit The sum of the digits is an even number. A number that fits this description is _____. Multiplication fact(s):

- One of the digits in this 2-digit number is 5, but the number is not divisible by 5. Nor is it divisible by 9. A number that fits this description is _____. Multiplication fact(s):
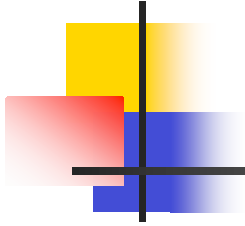
- How would you solve 14 x 8 mentally?

- Use an area model to show how 14 x 8 can be decomposed into 10 x 8 and 4 x 8.

- 14 x 8 = (10 x 8) + (4 x 8)

Boxes to multiply…

- Draw a rectangle to show 46 x 7 = 322

7

280

40

6

42

46 x 7 = (40 x 7) + (6 x 7)

=

280 + **42** = **322**

Navigations 3-5, Number and Operations, 2007

36
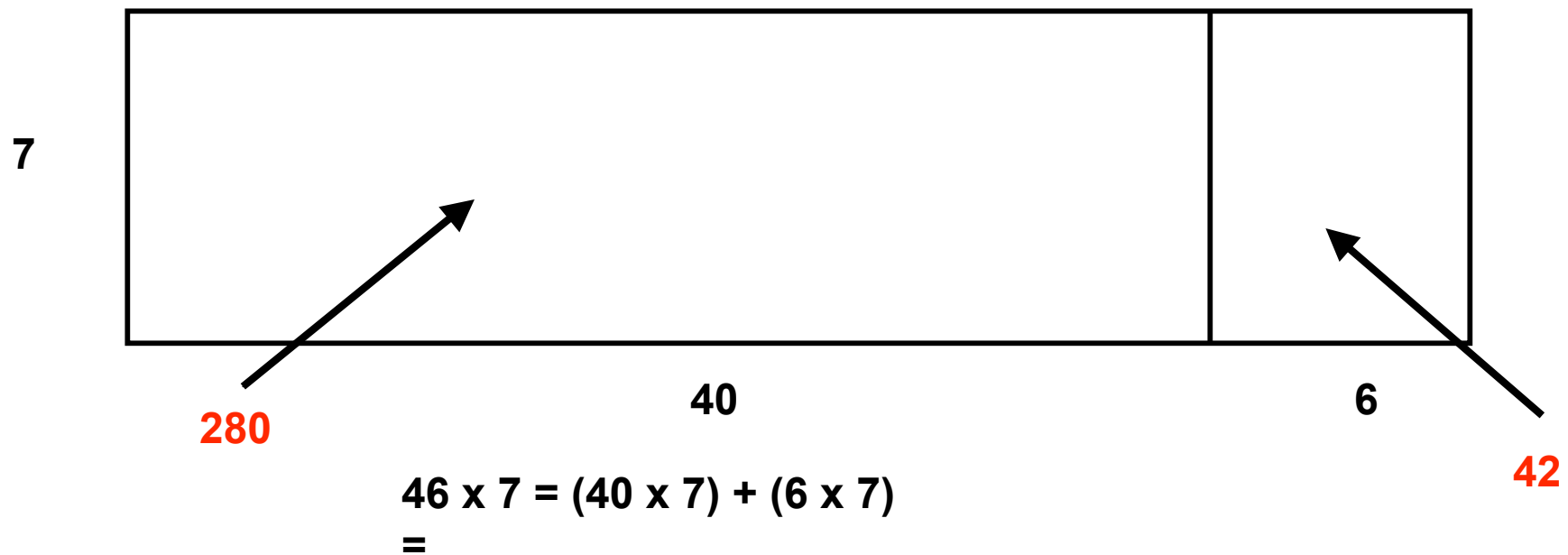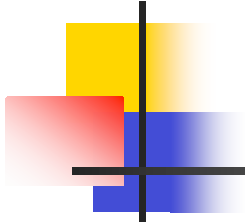
How about 45 x 23

$$45 \times 23$$

	40	5
20	40 x 20	5 x 20
3	40 x 3	5 x 3

Connections – Division & Mental Math

- 275 divided by 5
- Starter problem 250 ÷ 5

- Quinn found 77 beads in a drawer. He was using them to make bookmarks. If he used 5 beads for each bookmark. How many bookmarks could he make?
- Starter problem 50 ÷ 5
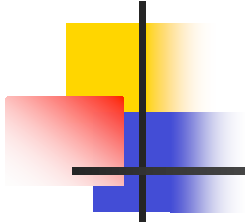
What pattern is being displayed?

What's the rule?

What's the graph look like?

In	Out
6	19
8	25
10	a
b	37

Estimation – Some Thoughts

- Estimating Magnitude – should begin early and occur often.

- Children are initially uncomfortable with computational estimation.

- The language of computational estimation is adult language. Children seem OK with such language as they grow – experientially.

Between - Density

- Name a number between 17 and 25.

- Name a number between 76 and 77.

- Name a number between 3.49 and 3.53.

- Name a number between 3.4 and 3.5.

- Name a number between 1/8 and 1/4.

Target Number

- Start number is 6
- Goal number is 420
- Write equations to show how you can get to the goal number.

- Start = 13; Goal = 100
- Start = 1/2 ; Goal = 5

Estimate or Exact?

- Your school bus number.
- When to leave for school in the AM.
- When a flight will leave the airport.
- Total bill at a restaurant.

- When do you estimate?
- When must you have an exact response?

How many digits in the answer?

- 174 + 689 =
- 134 + 989 =
- 1,246 − 348 =
- 874 − 567 =
- 12 x 48 =
- 12 x 336 =
- 2,344 ÷ 4 =

Reys

Think about this - A test!

- Four 2-digit numbers were added together.
  - The sum is 100
  - One of the addends is in the 20's.
  - One of the addends is in the 50's.
  - What can you say about the other two addends?

45

Ordering Fractions

Write these fractions in order from least to greatest. Tell how you decided.

- 5/3   5/6   5/5   5/4   5/8

- 7/8   2/8   10/8   3/8   1/8

Use Percent – Don't Wait!

- Put 2/3; 0.5 and 3/4 in order from smallest to largest.

- It's easy, 0.5 is 50% and 2/3 is 66%, and so it goes first 0.5, then 2/3 and then ¾ because that's 75%.*

*response by Andy in New Approaches to Teaching the Rational Number System by Joan Moss in How Students Learn: Mathematics in the Classroom, NRC, 2005.

Percent Benchmarks

0%

100%   50%   < 10%

About 25%   About 75%   About 90%

> 50%   < 50%

- Lefthanders in the room or class
- Once lived in New Jersey
- Been involved in education > 10 years
- People who were born in Pennsylvania

Missing Numbers

- What's my number?
- $2x + 7 = y$
- Rule: Double the number and add 7. What's the number if x =

<p style="text-align:center">10</p>
<p style="text-align:center">100</p>
<p style="text-align:center">0.1</p>
<p style="text-align:center">0.01</p>

Decimals - What Happens?

Number	x 0.05	x 0.48	x 0.9

100

60

24

?

- In general, what happens when you multiply a whole number by: 0.05; 0.48; 0.9?
- Begin thinking of 0.05 as 5% or nickel:dollar, etc.

50

Where's the decimal point go?

- 8.432 x 5.75 = 48484

- 3.044 x 16.5 = 50226

- 3.326 x 0.32 x 31.5 = 3352608

- 306.15 ÷ 75.4 = 40603448

Name that decimal!

- A decimal > 3 and < 4
- A decimal > 2.15
- A decimal < 3
- Two decimals whose sum = 1
- Three decimals whose sum < 0.8
- Four decimals whose sum = 2.35

And the equation is?

Start	Number of Operations	Total
5	two	13
36	three	100
¼	four	½

Today's Target is 36

- Try to make today's target by:
  - Adding 2 numbers
  - Finding the difference of 2 numbers
  - Multiplying 2 numbers
  - Adding 3 numbers
  - Multiplying 3 numbers
  - Multiplying and subtracting
  - YOUR own method!

McIntosh, Reys, Reys, and Hope (1997)

Algebra - Equations

54

Real Life Number Sense - Applications

Examples of Change

- At age 13, Jesse ran a mile in 5:40, how fast might he be able to run a mile at age 19?

- The drive to Williamsport took 25 minutes. How long will the trip take to get home?

- There were 7 people in the house at dinner time. How many people will be there for lunch?

Algebra – Change; Measurement

Right or Wrong – Why?

- Tom found the average weight of children in his 4th grade class was 196 pounds.

- Jack thought 7 x 89 was about 350.

- Joe is 9, he weighs 70 pounds. When he is 36 years old, will he weigh 4 times as much?

Buswell, et al (1955)

Estimation

- How many 1-digit numbers are there? 2-digit numbers? 3-digit numbers?

- The toll road is 243 miles long. If you traveled at a speed of 61 mph, about how many hours will you be on the toll road?

- The height of full grown human is about 21 times the length of the middle finger.

Elementary School Mathematics, Addison-Wesley, 1968

Are you sure?

Actual problem presented at a mathematics conference.

A dog traveled 15 meters per second. How far would the dog go in: a minute, a half-hour, an hour, a day?

Speeds of Some Animals

Cheetah	70 mph (65)*
Lion	50 mph
Zebra	40 mph
Rabbit	35 mph
"Super Dog"	33+ mph
Reindeer	32 mph
Elephant	25 mph
Chicken	8 mph

OK, what do we do about:

Time

- Where does this belong?
- Do we care about digital and analog time?
- Rich source of patterns and functions

Money

- This is hard for many children.
- What about models?
- Rich source of patterns and functions

What can you do in a minute?

- Sit-ups
- Listen to a song
- Finish my homework!
- Do a chore

- Wait a minute – really?

Just a Second...

- How many seconds do you spend at school each day?

- Describe what you were doing 1 million seconds ago.

- How old will you be and what do you expect to be doing 1 billion seconds from now?

Mann and Hartweg, **TCM**, 2005

What is your expected finishing time?

"Oh, about 2:45"

Time and Fractions

1/2 + 1/4 = 3/4; 6 + 3 = 9 of 12 or 3/4

5/6 - 1/2 = 1/3; 10 - 6 = 4 of 12 or 1/3

1/4 + 2/3 = 11/12

2 1/4 ÷ 3/4 =

1/3 x 7 = 2 1/3

Algebra – Models; Measurement

Liz's Pizza Palace

- At Liz's the cost of a large pizza is $8.00, but she always gives a $2.00 off coupon to teachers. On Tuesday's pizza is 25% of the regular price. Heather, a teacher, has a coupon. The coupon can be used on Tuesday's.

- Does it make a difference when the value of the coupon is deducted from the price of the pizza?

TRUE STORY!

Concluding Thoughts

- Number sense is elusive
- Number sense should be nurtured – every day!
- A sense of number breeds confidence.
- Number sense is not the final chapter in a 12 chapter book!
- Numb3rs are everywhere!