Common Core State Standards: Mathematics

OC Common Core Kick-Off
August 16, 2011

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&
Project Director
Elementary Math Specialists & Teacher Leaders Project

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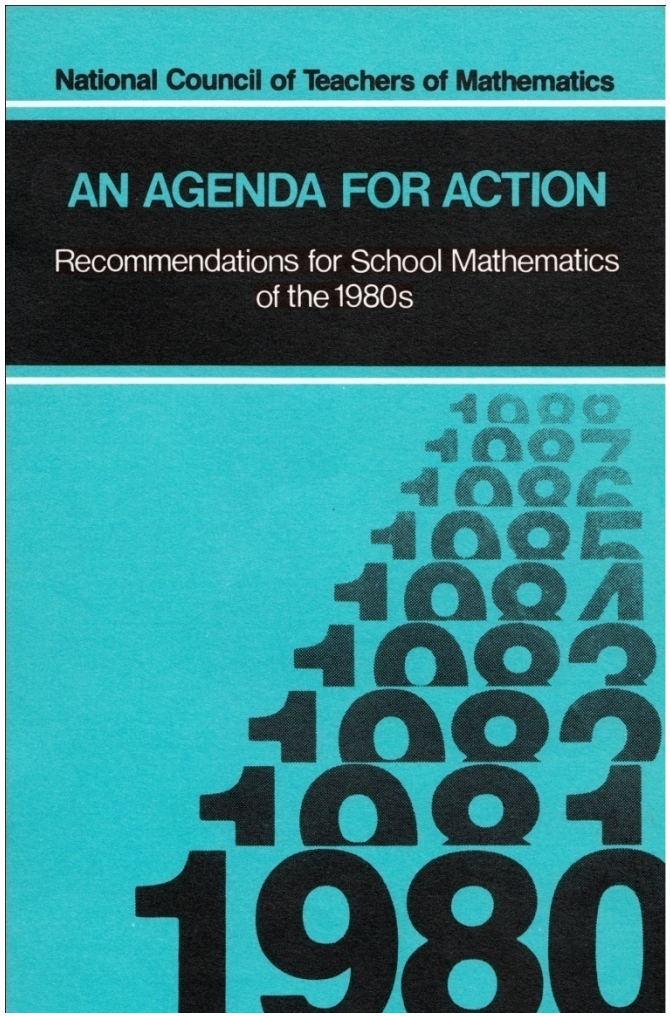
An Agenda for Action



- **Recommendation 1**: Problem Solving must be the Focus of School Mathematics in the 1980s;
- **Recommendation 2**: The Concept of Basic Skills in Mathematics Must Encompass More Than Computational Facility;
- **Recommendation 5:** The success of mathematics programs and student learning be evaluated by a wider range of measures than conventional testing;
- **Sound familiar…, and then**

NCTM, 1980

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Remember

K-4

1. ***Mathematics as Problem Solving***
2. ***Mathematics as Communication***
3. ***Mathematics as Reasoning***
4. ***Mathematical Connections***
5. Estimation
6. Number Sense and Numeration
7. Concepts of Whole Number Operations
8. Whole Number Computation
9. Geometry and Spatial Sense
10. Measurement
11. Statistics and Probability
12. Fractions and Decimals
13. Patterns and Relationships

5-8

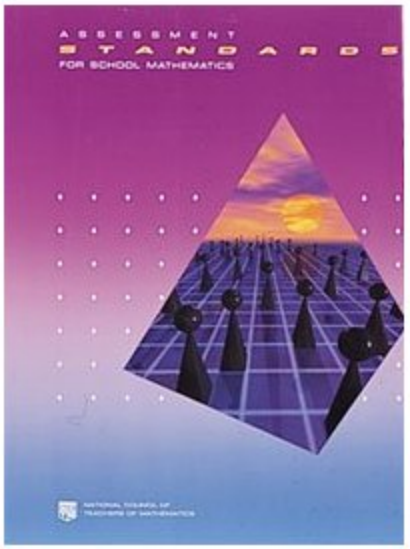
1. ***Mathematics as Problem Solving***
2. ***Mathematics as Communication***
3. ***Mathematics as Reasoning***
4. ***Mathematical Connections***
5. Number and Number Relationships
6. Number Systems and Number Theory
7. Computation and Estimation
8. Patterns and Functions
9. Algebra
10. Statistics
11. Probability
12. Geometry
13. Measurement

NCTM, 1989

The triology

1989

1991



1995

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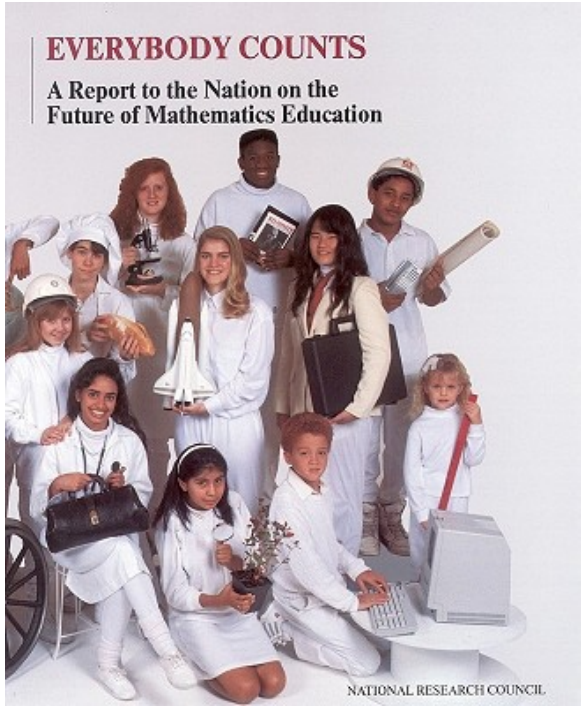
Then what…

- Universal acceptance
- Everybody Counts – companion document
- NSF supported curriculum projects
  - Middle
  - Elementary
  - High School
- Curriculum Center Projects
  - Elementary
  - Middle
  - High
  - K-12
- **Mid to Late 1990's**
  - **Honeymoon Over – Math Wars…**



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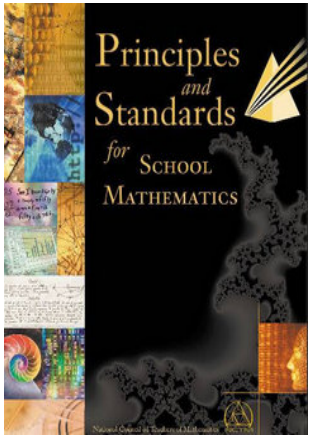

Next Step…

Pre-K-2; 3-5; 6-8; 9-12

- Number and Operations
- Algebra
- Geometry
- Measurement
- Data Analysis and Probability
- *Problem Solving*
- *Reasoning and Proof*
- *Communication*
- *Connections*
- *Representation*

NCTM, 2000

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and then…the focus and coherence thing…

NCTM, 2006

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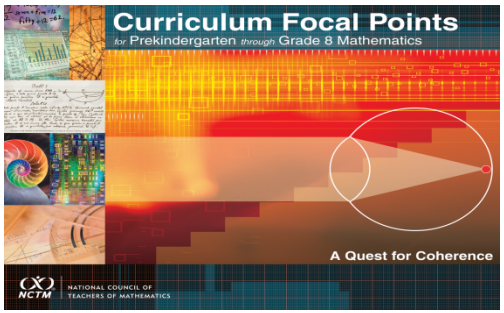
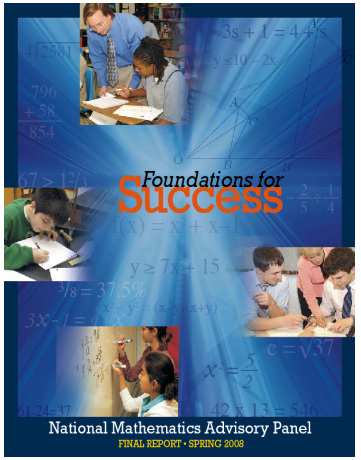
Why Focus and Coherence?

- Long lists of state learning expectations
- "Mile wide, inch deep"
- Mobility
- **International Comparisons**
- **Common Curriculum Clamoring – both sides of the aisle – *really!***
- **National Math Advisory Panel Recommendations**





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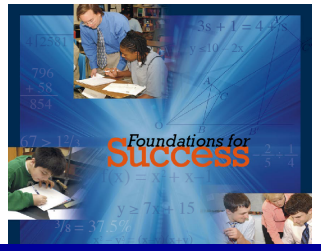
Driving the CCSS



LEADERSHIP, ADVOCACY, SERVICE
CCSSO
THE COUNCIL OF CHIEF STATE SCHOOL OFFICERS

NATIONAL GOVERNORS ASSOCIATION

OECD
PISA

IEA
TIMSS
·2007·

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One year ago…

"This will change your life and what you do as a mathematics specialist…"

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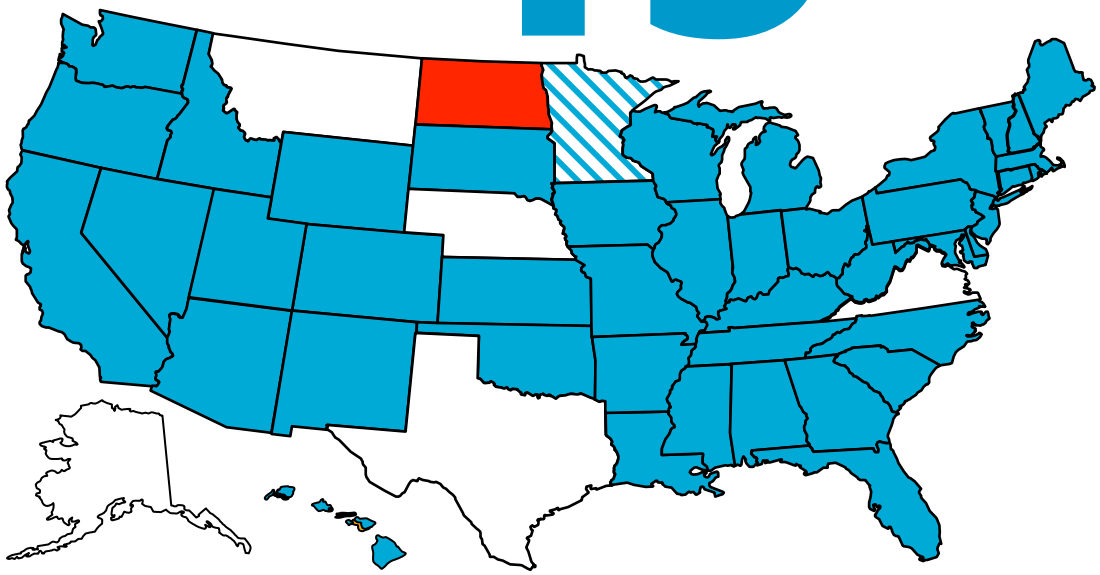
AND Now…

Common Core State Standards Adoptions

45 states

State adopted standards in only one subject



FEBRUARY
10 Kentucky

MAY
12 West Virginia
20 Hawaii
25 Maryland

JUNE
2 Wisconsin
3 North Carolina
4 Utah
7 Ohio
15 Michigan
15 Missouri
16 New Jersey
16 Wyoming
18 Nevada
24 Illinois
14 Oklahoma

JULY
1 Louisiana
1 Pennsylvania
1 Rhode Island
7 Connecticut
8 Georgia
8 New Hampshire
12 Arkansas
14 South Carolina
19 New York
19 Washington
21 Massachusetts
21 Washington, D.C.
27 Florida
29 Iowa
30 Tennessee

AUGUST
2 Colorado
2 California
3 Indiana
17 Vermont
19 Delaware

SEPTEMBER
27 Minnesota

OCTOBER
12 Kansas
19 New Mexico
28 Oregon

NOVEMBER
17 Idaho
18 Alabama
29 South Dakota

APRIL 2011
1 Maine

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Source: Education Week

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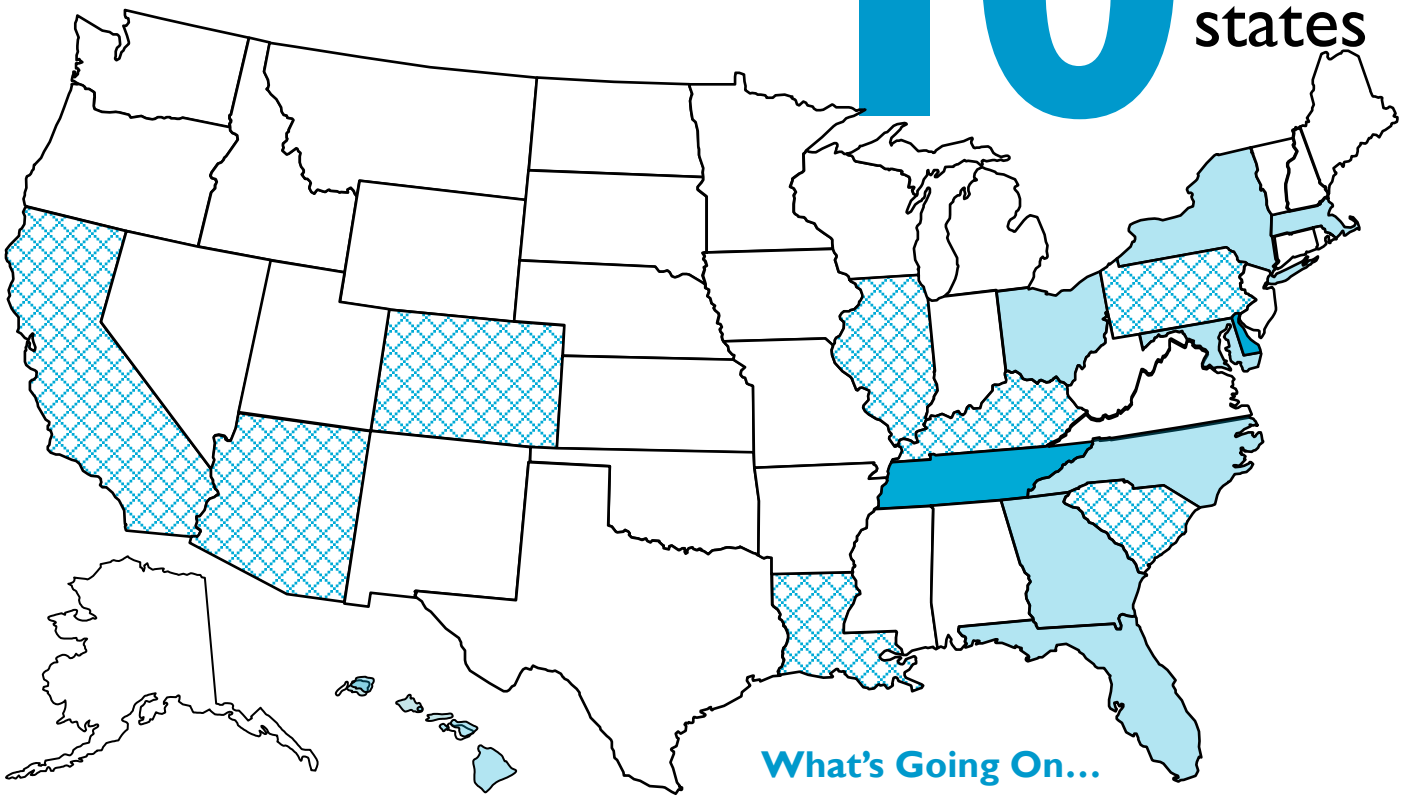
Race to the Top Competition Results

10 states



Phase I Winners

Phase II Winners

Submitted, No Award

What's Going On…

New Race to the Top:
$500M for Early Education
$200M for Round 2 Runners-Up

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Math Groups Support Common Standards

TO THE EDITOR:

The final set of common academic standards released June 2 by the Common Core State Standards initiative are a welcome milestone in the standards movement that began more than 20 years ago when the National Council of Teachers of Mathematics published its "Curriculum and Evaluation Standards for School Mathematics." The new common standards provide the foundation for more-focused and coherent instructional materials and assessments that measure students' understanding of mathematical concepts and acquisition of fundamental reasoning habits, in addition to fluency with math skills. …

Letter co-signed by NCTM, NCSM, AMTE, ASSM; June 14, 2010
Additional efforts forthcoming by NCTM and NCSM – stay tuned

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Let's take a look…

The Starting Point…

NCTM Processes	*CCSS – Standards for Mathematical Practice*	*Adding it Up – Strands of Mathematical Proficiency*
Problem Solving	***Make sense of problems and persevere in solving them.***	Strategic competence
Reasoning and Proof	***Reason abstractly and quantitatively.***	Adaptive reasoning
Reasoning and Proof	***Construct viable arguments and critique the reasoning of others.***	Adaptive reasoning
Connections	***Model with mathematics.***	Strategic competence
Representation	***Use appropriate tools strategically.***	Strategic competence Conceptual understanding
Communication	***Attend to precision.***	Procedural fluency.
Connections	***Look for and make use of structure.***	Strategic competence
Reasoning and Proof	***Look for and express regularity in repeated reasoning.***	Adaptive reasoning
		*Productive disposition

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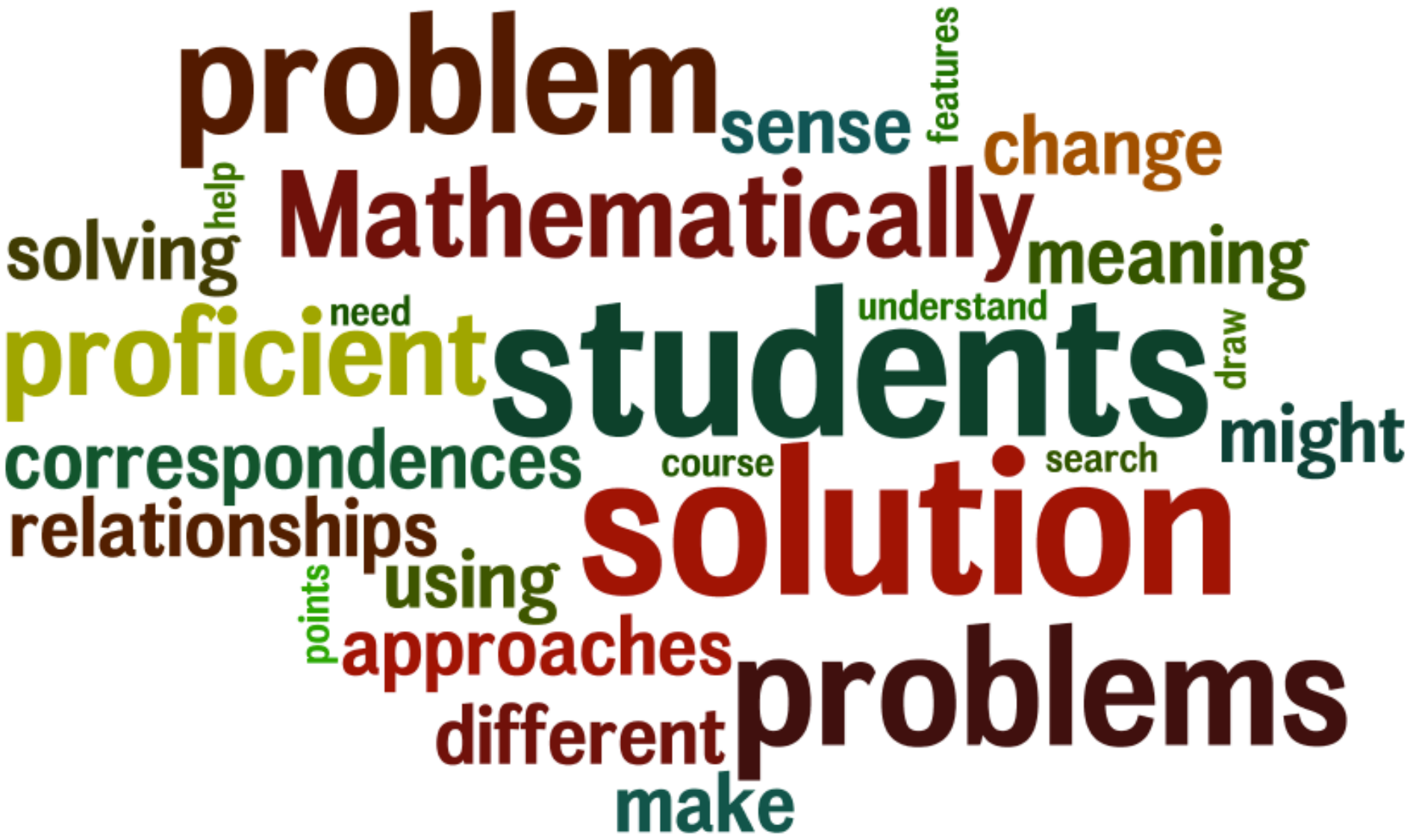
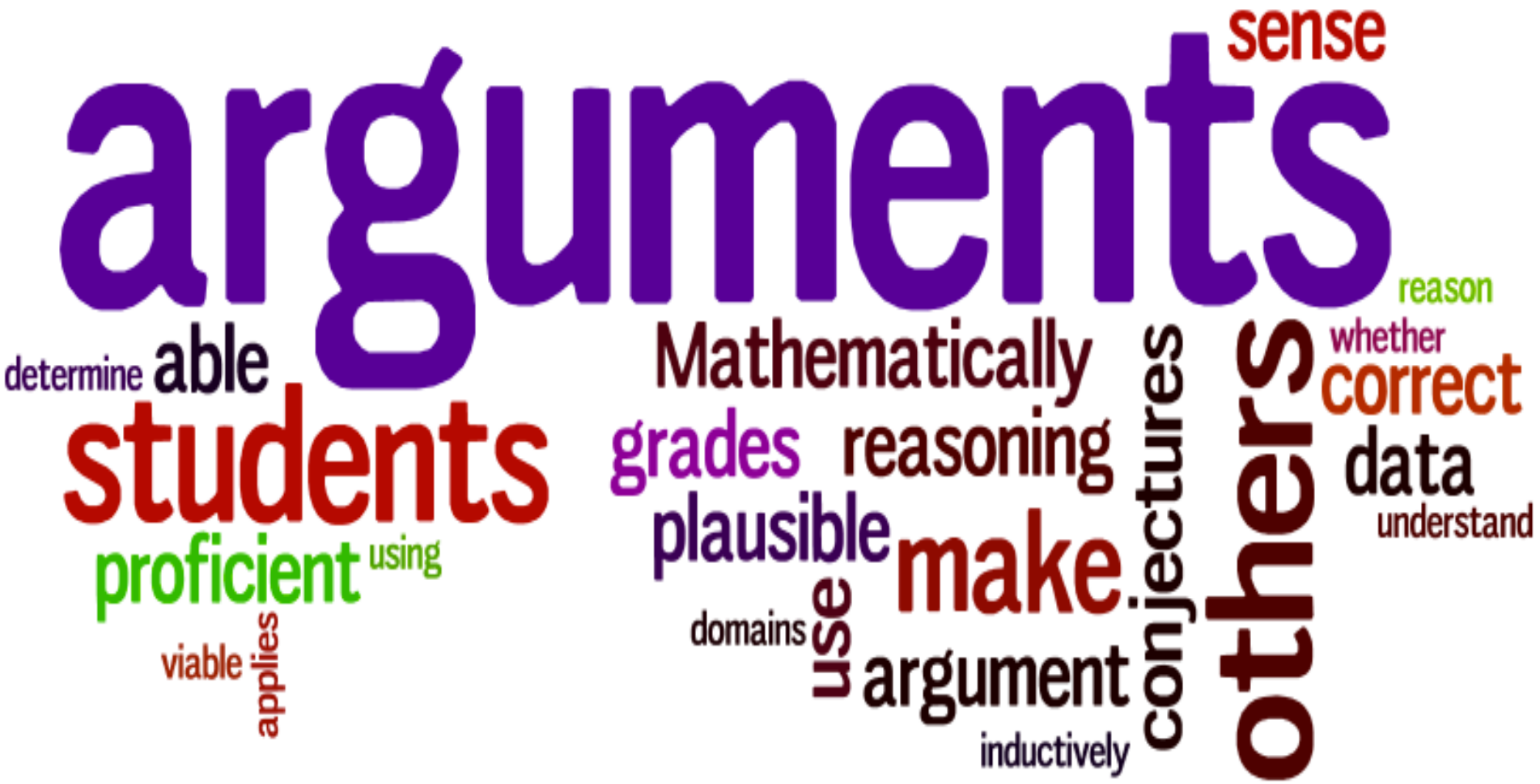
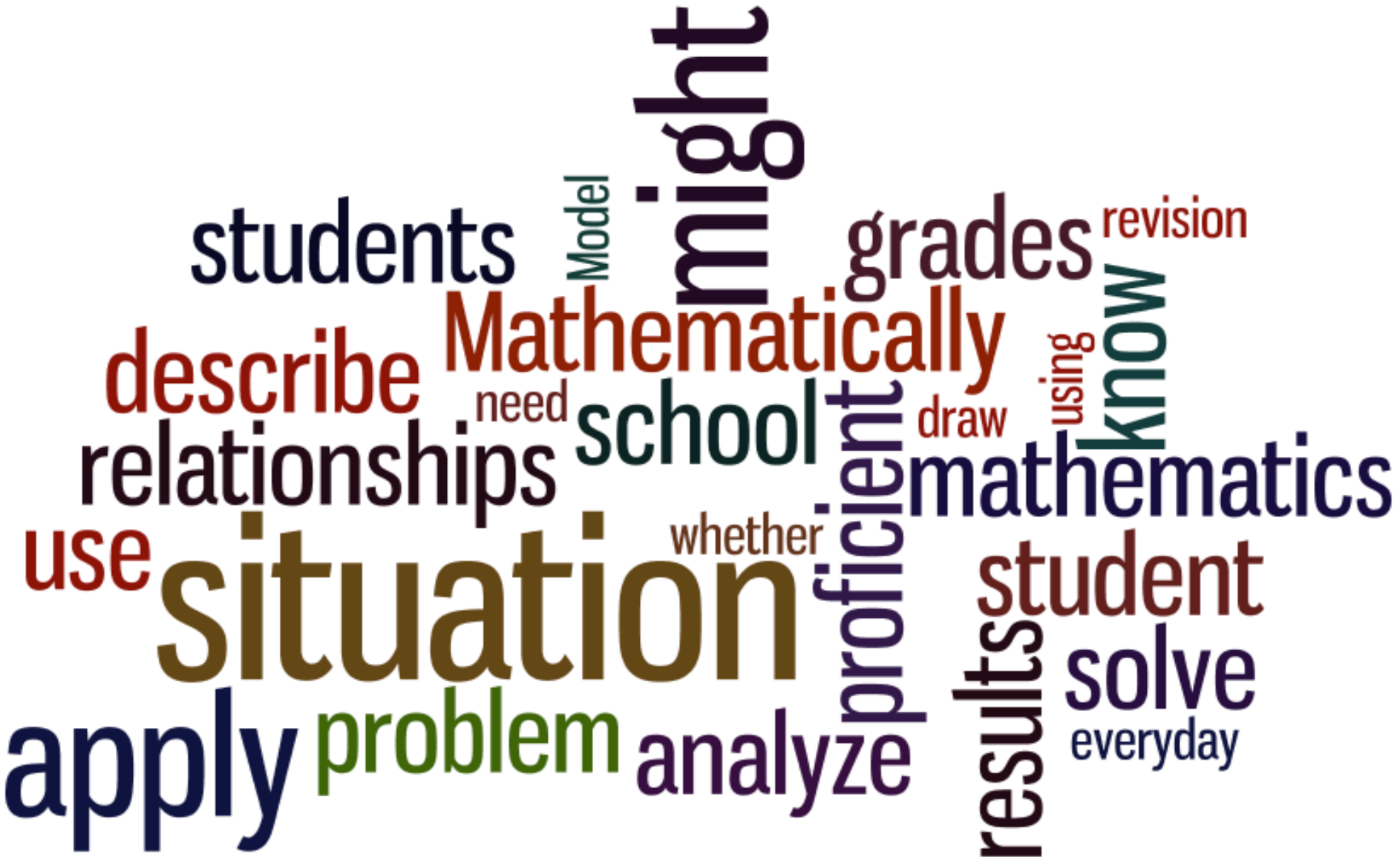
1. Make sense of problems and persevere in solving them



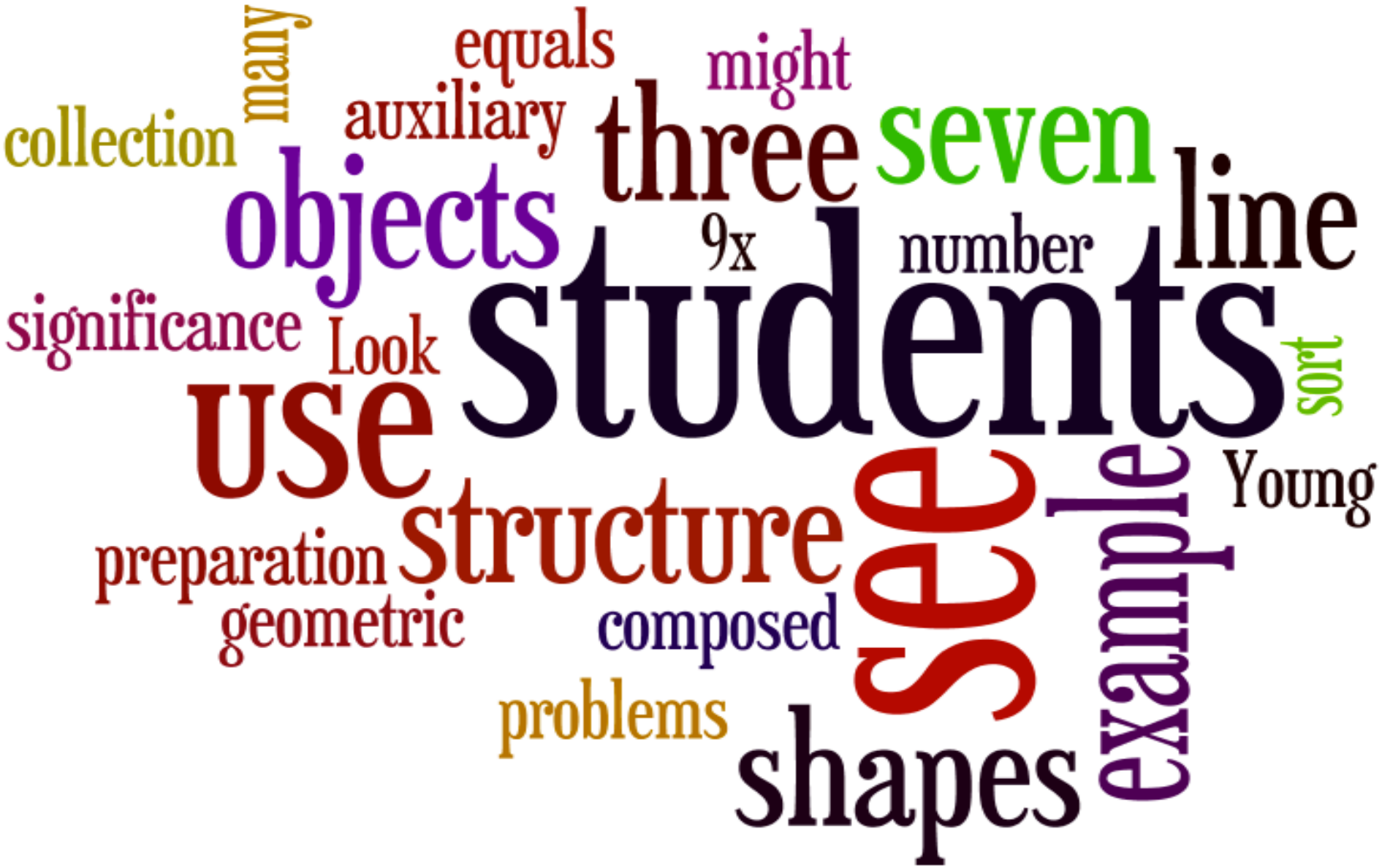
3. Construct viable arguments and critique the reasoning of others



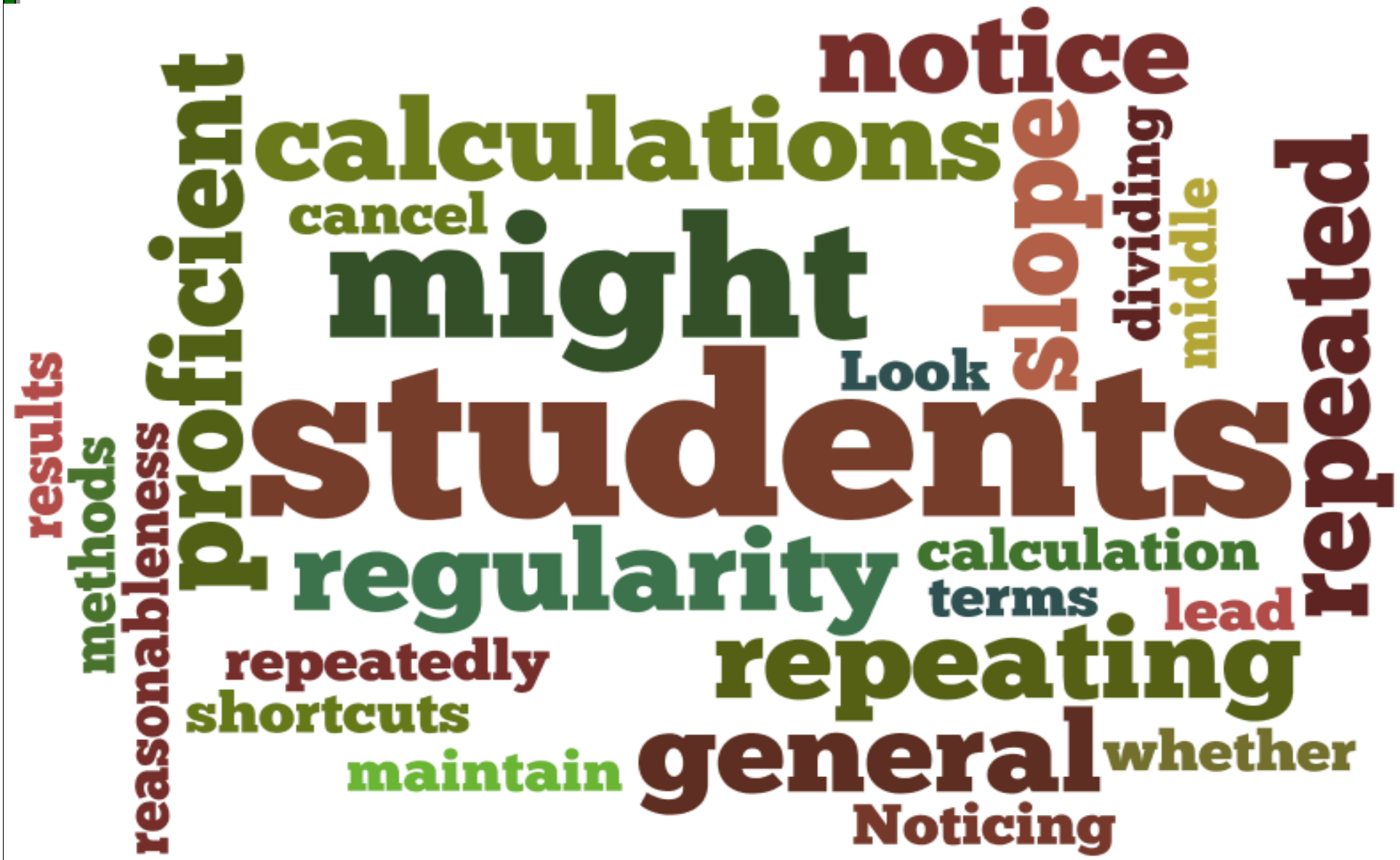
4. Model with mathematics



7. Look for and make use of structure



8. Look for and express regularity in repeated reasoning



Look For's

Make sense of problems and persevere in solving them (e.g.):

1.Students: Are actively engaged in solving problems

2.Teacher: Provides time for and facilitates the discussion of problem solutions

Reason abstractly and quantitatively (e.g.):

1.Students: Use varied representations and approaches when solving problems

2.Teacher: Provides a range of representations of mathematical ideas and problem situations and encourages varied solution paths

Construct viable arguments and critique the reasoning of others (e.g.):

1.Students: Understand and use prior learning in constructing arguments

2.Teacher: Provides opportunities for students to listen to or read the conclusions and arguments of others

Model with mathematics (e.g.):

1.Students: Apply mathematics learned to problems they solve and reflect on results

2.Teacher: Provides a variety of contexts for students to apply the mathematics learned

Use appropriate tools strategically (e.g.):

1. Students: Use technological tools to deepen understanding
2. Teacher: Uses appropriate tools (e.g. manipulatives) instructionally to strengthen the development of mathematical understanding

Attend to Precision (e.g.):

1.Students: Based on a problem's expectation, students calculate with accuracy and efficiency.

2.Teacher: Emphasizes the importance of mathematical vocabulary and models precise communication.

Look for and make use of structure (e.g.):

1.Students: Look for, develop, and generalize arithmetic expressions

2.Teacher: Provides time for applying and discussing properties

Look for and express regularity in repeated reasoning (e.g.):

1.Students: Use repeated applications to generalize properties

2.Teacher: Models and encourages students to look for and discuss regularity in reasoning

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Construct viable arguments and critique the reasoning of others.

- Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments.
- They justify their conclusions, communicate them to others, and respond to the arguments of others.
- Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.
- Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades.
- Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

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adapted from the Mathematical Practices

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from the ems&tl look for's

- **Students**: Understand and use prior learning in constructing arguments

- **Teacher**: Provides opportunities for students to listen to or read the conclusions and arguments of others

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- Can you represent the area of a rectangle that measures 5” by 8”? Use grid paper of tiles for your representation.

- What if the rectangle measured 5.5” by 7.5”? How would you represent the area? Would the area for this rectangle be the same as for the rectangle above?

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Before you can 'look for'

- Teacher content background

- Teacher pedagogical background
- Issues:
  - Is the problem appropriate?
    - *Does it build from prior experiences?*
  - Representations appropriate?
  - Access to materials?

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Before you can 'look for'

- Instructional Issues
  - Do the students have enough TIME to consider the problems, solve them, and discuss them – including similarities and differences?

- Classroom norms for:
  - Discussing?
  - Engaging others – engaging all?
  - Presenting and accepting arguments?

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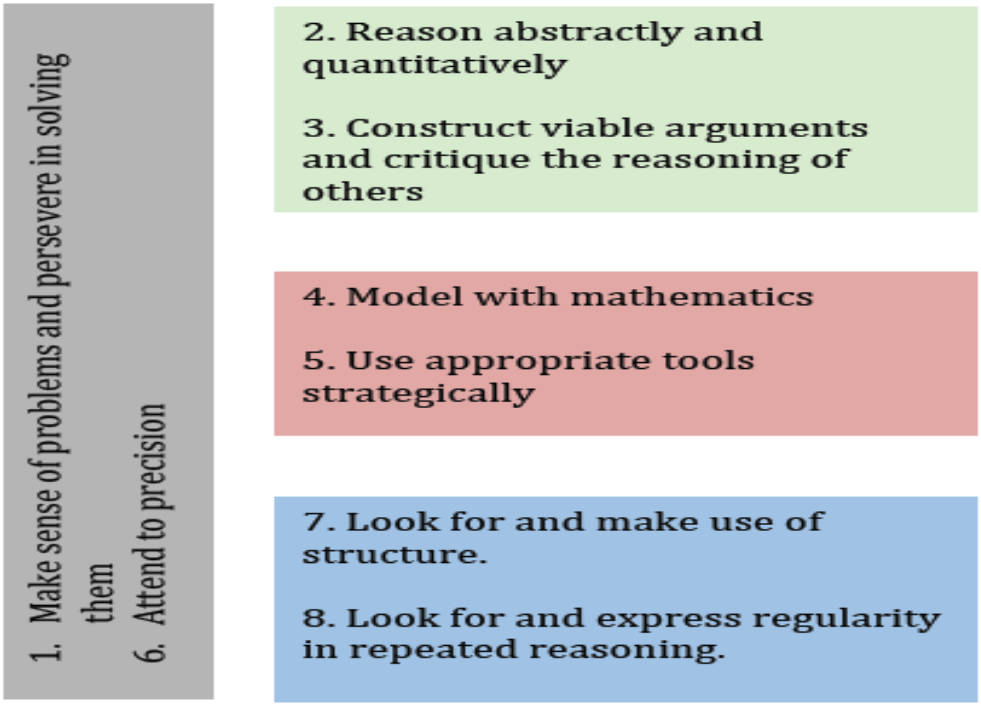
Another Look



Bill McCallum's blog!

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Content - now…

Grades K-2	*Grades 3-5*
• **Counting and Cardinality (K only)**	• Operations and Algebraic Thinking
• Operations and Algebraic Thinking	• Number and Operations in Base Ten
• Number and Operations in Base Ten	• **Number and Operations – Fractions**
• Measurement and Data	• Measurement and Data
• Geometry	• Geometry

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CCSSO, 2010

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and…

Grades 6, 7	*Grades 8*
• **Ratios and Proportional Relationships** • The Number System • Expressions and Equations • Geometry • Statistics and Probability	• The Number System • Expressions and Equations • **Functions** • Geometry • Statistics and Probability

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CCSSO, 2010

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and…

High School Conceptual Categories

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability
- **Note on courses and transitions**: course sequence, K-7 standards prepare students for Algebra I in grade 8, etc.

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CCSSO, 2010

Algebra for EVERY Student

- **Algebra as…**
  - Precursor
  - A way to extend arithmetic
  - Generalized arithmetic
  - The first serious mathematics course for many
  - Gatekeeper
  - …

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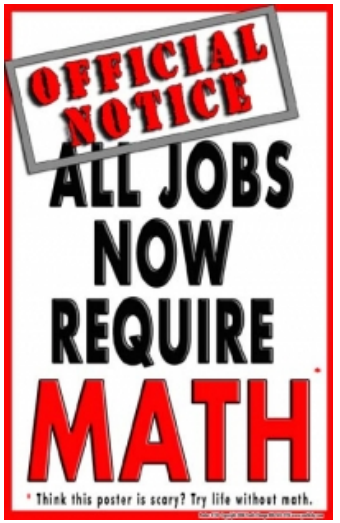
and Algebra

- **Grades 3-5**
  - Operations and Algebraic Thinking
    - Grade 3
      - Determine the unknown whole number; 5 = ■ ÷ 3
    - Grade 4
      - Gain familiarity with factors and multiples (prime, composite)
    - Grade 5
      - Write and interpret numerical expressions
- **Grade 6-8**
  - Ratios and Proportional Relationships
  - The Number System
  - Expressions and Equations
  - Functions (grade 8)



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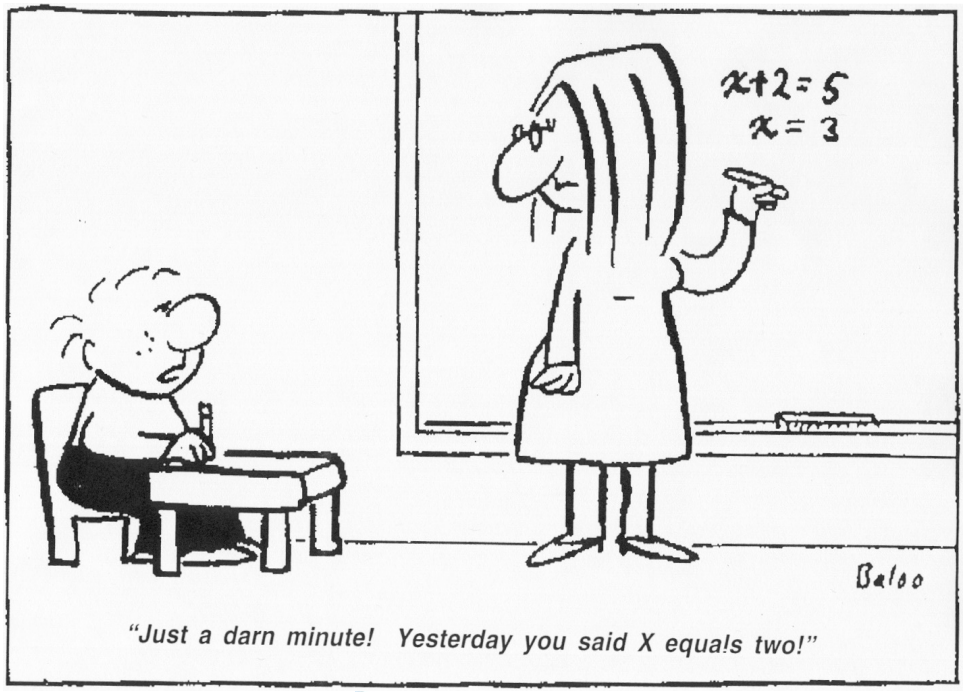
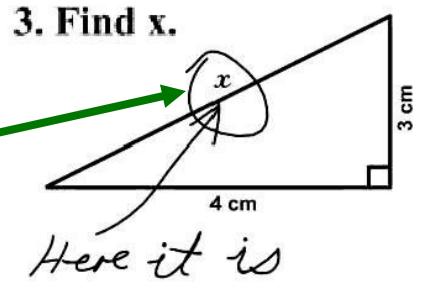
What Algebra? When?

- Grade level?
- Background?
- Who's teaching?
- The Misplaced Math Student – Lost in 8th Grade Algebra…





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- Hey frank,
- The problem asked to write an equation that represented 4 more than 3 times a number. He wrote

  4 + (3 x n) and she was looking for 3n+4. She said the 3n should start the answer and therefore his answer was incorrect. I did write her a note today letting her know that based on the rules of the order of operations, parentheses always take precedence. So left to right only comes into play after the parentheses have been worked. I didn't get a response, but I'm sure I made her last day of school.
- And then…

Oh my…

"Say something about the Order of Operations"

Like numerical order and you do it in a certain order.

Way to do things like multiplying problems; different signs are presented and the order your need to do them in.

First make sure they are clean, then take their clothes off, then cut, then sew up, then back to their room.

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A glimpse…probably not fair

	Cluster Expectations
K	24
1	23
2	27
3	30
4	34
5	34
6	43
7	44
8	33

- Don't let the number of understandings and skills be the whole story…

Less is more!?

Taking a chance…

Another look at content emphasis

Domains/ Grades	K	1	2	3	4	5	Totals
Counting and Cardinality	9						5%
Operations and Algebraic Thinking	5	8	4	9	5	3	20%
Number and Operations in Base Ten	1	8	10	3	6	8	21% (K-5); 17% (3-5)
Number and Operations - Fractions				7	12	11	31% (3-5)
Measurement and Data	3	4	10	12	8	8	26%
Geometry	6	3	3	2	3	4	12%
Totals	24	23	27	30	34	34	172

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Grades K-5

Take a Chance…

NOTE: Please consider this table as a 'for discussion ONLY' example of the impact of the CCSS. The totals above are only a count of the standards (or sub-standards) within a cluster, there is NO attempt here to consider weight/emphasis/time needed for particular standards, which is another AND VERY IMPORTANT consideration.

BUT, Think about:

- Number and Operations in Base Ten and Fractions – 48% of grades 3-5.
- Number related domain emphasis (operations and algebraic thinking, number and operations in base ten, and number and operations – fractions):
  - 63% in grade 3
  - 68% in grade 4
  - 65% in grade 5
- What do YOU see? What do YOU wonder about?

Domains Grades	6	7	8	Totals
Ratios and Proportional Relationships	7	6		15% (grades 6, 7)
The Number System	13	9	2	20%
Expressions and Equations	11	5	11	23%
Functions			5	15% (grade 8)
Geometry	4	6	11	18%
Statistics and Probability	8	11	4	19%
Totals*	44	43	33	120

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Grades 6-8

Take a Chance…

NOTE: Same qualification as with grades K-5.

BUT, Think about:

- Ratio and proportional relationships, the number system, expressions and equations – 72% of grade 6.
- Algebra related domain emphasis (ratio and proportional relationships; number system, expressions and equations, functions):
  - 72% in grade 6
  - 45% in grade 7
  - 55% in grade 8
- Statistics and probability emphasis:
  - 19% in grade 6
  - 25% in grade 7
  - 12% in grade 8
- What do *YOU* see? What do *YOU* wonder about?

Moving Forward

- *Transition to Implementation**
- *What's important?*
  –three considerations…

**what's YOUR implementation plan?*

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Grade 7 Critical Areas

1. Developing understanding of and applying proportional relationships;

2. Developing understanding of operations with rational numbers and working with expressions and linear equations;

3. Solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume;

4. Drawing inferences about populations based on samples.

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Curriculum Focal Points and Connections for Grade 7

The set of three curriculum focal points and related connections for mathematics in grade 7 follow. These topics are the recommended content emphases for this grade level. It is essential that these focal points be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

Grade 7 Curriculum Focal Points	Connections to the Focal Points
Number and Operations **and** ***Algebra*** **and** ***Geometry:*** **Developing an understanding of and applying proportionality, including similarity** Students extend their work with ratios to develop an understanding of proportionality that they apply to solve single and multistep problems in numerous contexts. They use ratio and proportionality to solve a wide variety of percent problems, including problems involving discounts, interest, taxes, tips, and percent increase or decrease. They also solve problems about similar objects (including figures) by using scale factors that relate corresponding lengths of the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and identify the unit rate as the slope of the related line. They distinguish proportional relationships ($y/x = k$, or $y = kx$) from other relationships, including inverse proportionality ($xy = k$, or $y = k/x$).	***Measurement*** and ***Geometry:*** Students connect their work on proportionality with their work on area and volume by investigating similar objects. They understand that if a scale factor describes how corresponding lengths in two similar objects are related, then the square of the scale
Measurement **and** ***Geometry*** **and** ***Algebra:*** **Developing an understanding of and using formulas to determine surface areas and volumes of three-dimensional shapes** By decomposing two- and three-dimensional shapes into smaller, component shapes, students find surface areas and develop and justify formulas for the surface areas and volumes of prisms and cylinders. As students decompose prisms and cylinders by slicing them, they develop and understand formulas for their volumes (*Volume = Area of base × Height*). They apply these formulas in problem solving to determine volumes of prisms and cylinders. Students see that the formula for the area of a circle is plausible by decomposing a circle into a number of wedges and rearranging them into a shape that approximates a parallelogram. They select appropriate two- and three-dimensional shapes to model real-world situations and solve a variety of problems (including multistep problems) involving surface areas, areas and circumferences of circles, and volumes of prisms and cylinders.	and b a… …dents continue to develop their unders… …ultiplication and division and the struc… …mbers by determining if a counting number grea… …1 is a prime, and if it is not, by factoring it into a p… …t of primes.
Number and Operations **and** ***Algebra:*** **Developing an understanding of operations on all rational numbers and solving linear equations** Students extend understandings of addition, subtraction, multiplication, and division, together with their properties, to all rational numbers, including negative integers. By applying properties of arithmetic and considering negative numbers in everyday contexts (e.g., situations of owing money or measuring elevations above and below sea level), students explain why the rules for adding, subtracting, multiplying, and dividing with negative numbers make sense. They use the arithmetic of rational numbers as they formulate and solve linear equations in one variable and use these equations to solve problems. Students make strategic choices of procedures to solve linear equations in one variable and implement them efficiently, understanding that when they use the properties of equality to express an equation in a new way, solutions that they obtain for the new equation also solve the original equation.	***Data Analysis:*** Students use proportions to make estimates relating to a population on the basis of a sample. They apply percentages to make and interpret histograms and circle graphs. ***Probability:*** Students understand that when all outcomes of an experiment are equally likely, the theoretical probability of an event is the fraction of outcomes in which the event occurs. Students use theoretical probability and proportions to make approximate predictions.

Data Analysis: Students use proportions to make estimates relating to a population on the basis of a sample. They apply percentages to make and interpret histograms and circle graphs.

TWO

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Understanding

4.NBT

- Generalize place value ***understanding*** for multi-digit whole numbers.
- Use place value ***understanding*** and properties of operations to perform multi-digit arithmetic.

4.NF

- Extend ***understanding*** of fraction equivalence and ordering.
- Build fractions from unit fractions by applying and extending previous ***understandings*** of operations on whole numbers.
- Understand decimal notation for fractions and compare decimal fractions.

4.MD

- Geometric measurement: ***understand*** concepts of angle and measure angles.

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Understanding

6.RP

- ***Understand*** ratio concepts and use ratio reasoning to solve problems.

6.NS

- Apply and extend previous ***understandings*** of multiplication and division to divide fractions by fractions.
- Apply and extend previous ***understandings*** of numbers to the system of rational numbers.

6.EE

- Apply and extend previous ***understandings*** of arithmetic to algebraic expressions.

6.SP

- Develop ***understanding*** of statistical variability.

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Go to CCSS, pages 42-45

Representation

- 3.NF.2 – Understand a fraction as a number on the number line; ***represent fractions on a number line diagram.***
- 4.NBT.5 – Multiply a whole number…Illustrate and explain…by ***using equations, rectangular arrays, and/or area models.***
- 5.MD.4 – Measure volumes by counting unit cubes, ***using cubic cm, cubic in, cubic ft, and improvised units.***
- 6.RP.3 – Use ratio and rate reasoning…by reasoning about ***tables of equivalent ratios, tape diagrams, double line diagrams or equations.***
- 8.FF.2 – Compare properties of two ***functions…represented in a different way (algebraically graphically, numerically in tables or by verbal descriptions).***

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COMMON CORE
STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

Major Point

- Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?* (4.NF.4c)

- **Understanding + Representations = Time; Stuff; Depth**

Conceptual understanding is NOT an option,

it's an expectation!

THREE

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COMMON CORE STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

Begin to discuss…

- What's familiar?
- What's new? What's challenging?
- Unpacking and Emphasis?
- Really important: "A lack of ***understanding*** effectively prevents a student from engaging in the mathematical practices." (p. 8)

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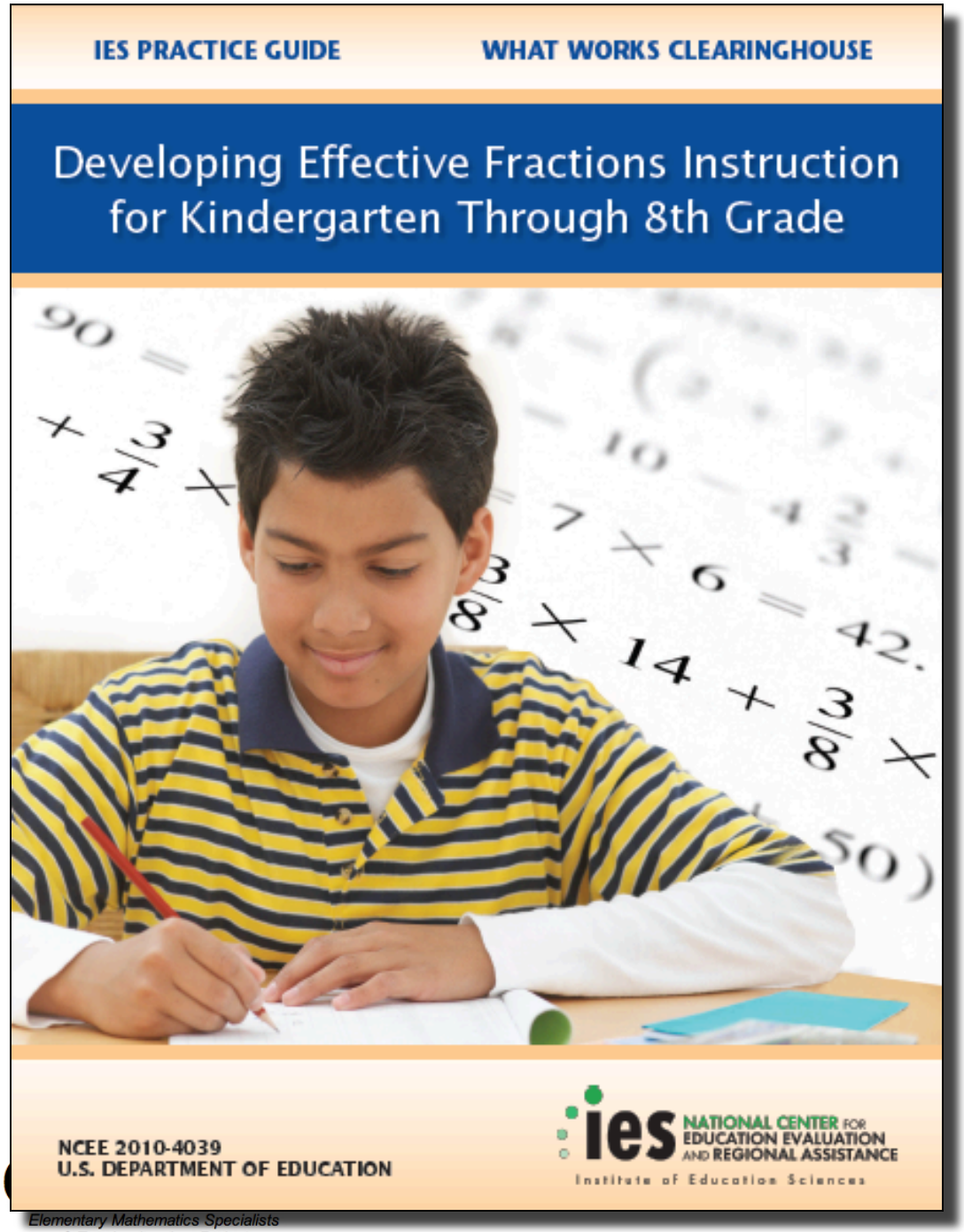
An example



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Panel

Robert Siegler (Chair)
Carnegie Mellon University

Thomas Carpenter
University of Wisconsin-Madison

Francis (Skip) Fennell
McDaniel College

David Geary
University of Missouri at Columbia

James Lewis
University of Nebraska-Lincoln

Yukari Okamoto
University of California-Santa Barbara

Laurie Thompson
Elementary Teacher

Jonathan (Jon) Wray
Howard County (MD) Public Schools

Staff

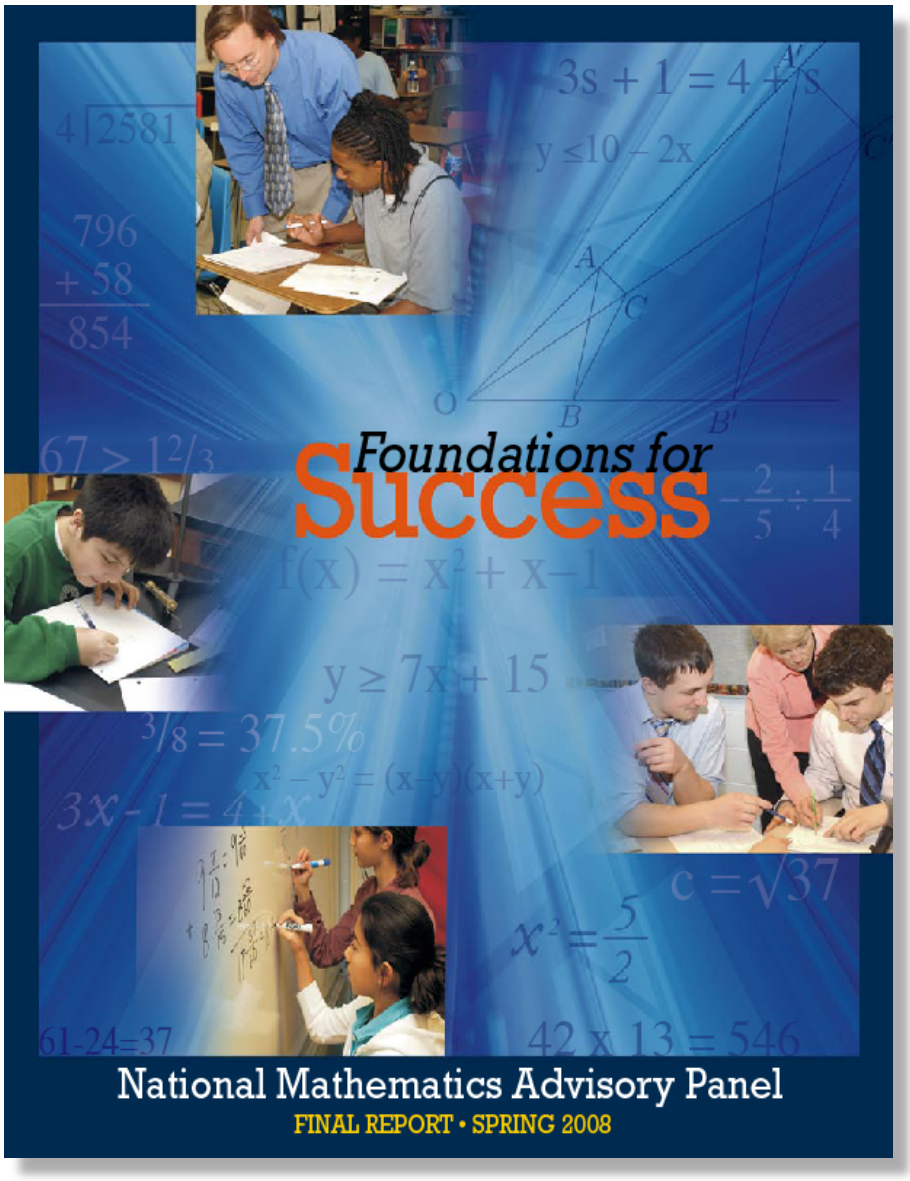
Jeffrey Max
Andrew Gothro
Sarah Prenovitz
Mathematical Policy Research

Project Officer - **Susan Sanchez**
Institution of Education Sciences (IES)

How did this get started…?

Fraction issues…

- Conceptual Knowledge and Skills
- Learning Processes
- Assessment
- Survey of Algebra Teachers



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Why are fractions so difficult?

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Recommendation 1

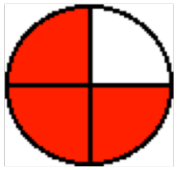
1. An early example: If we cut this cake so that you and two friends could share it, what would the slices look like? How can we talk about and write how much of the cake you will each get? (Circular or rectangular regions)
2. Would you rather share your favorite pizza with 3 other people or with 7 other people? (McNamara and Shaughnessy, ***Beyond Pizzas and Pies***, 2010)
3. How can you share 8 cookies with the four children in your family? Can you make a drawing to show me how you would do this? (Tiles or counters)
   - **Extending** - What if you had 10 cookies; now how would you share them with the 4 children in your family?
4. If you had 13 cookies to share among 4 friends, how many cookies would each person get? Would this be *more* or *less* than if you shared 12 cookies? (Tiles or counters)

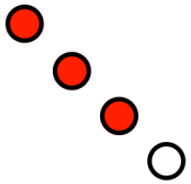
Recommendation 1

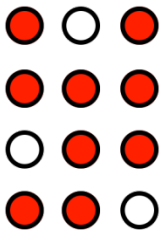
Think About

5. What about 6 people sharing 5 cookies? How much for each person?
6. Sharing 12 cards with 5 people; Sharing 5 cards with 12 people – how are these approached differently?
7. Each delivery is 3 miles. There were 7 deliveries. How many miles were traveled? *How would you represent this?*
8. Connections to CCSS (Grades 1, 2, 6, and 7)

Thinking about ¾…

a) 

b) 

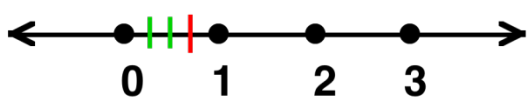
c) 

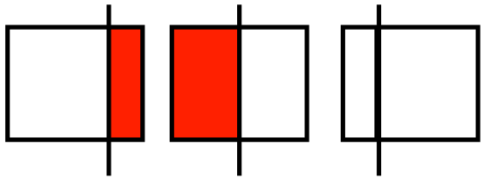
d) How many 4's are there in 3?

e) 18 crayons out of a box of 24

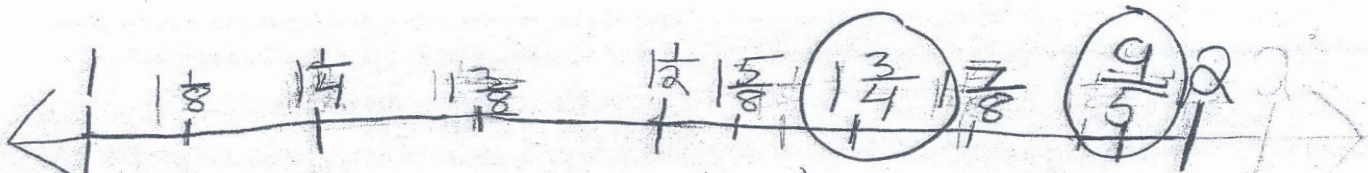
f) .75

g) I want to share 3 bottles of soda equally among 4 people. How much will each person get?

h) 

i) 

1) Draw a number line and show where to place the fraction 9/5.
Explain your thinking.



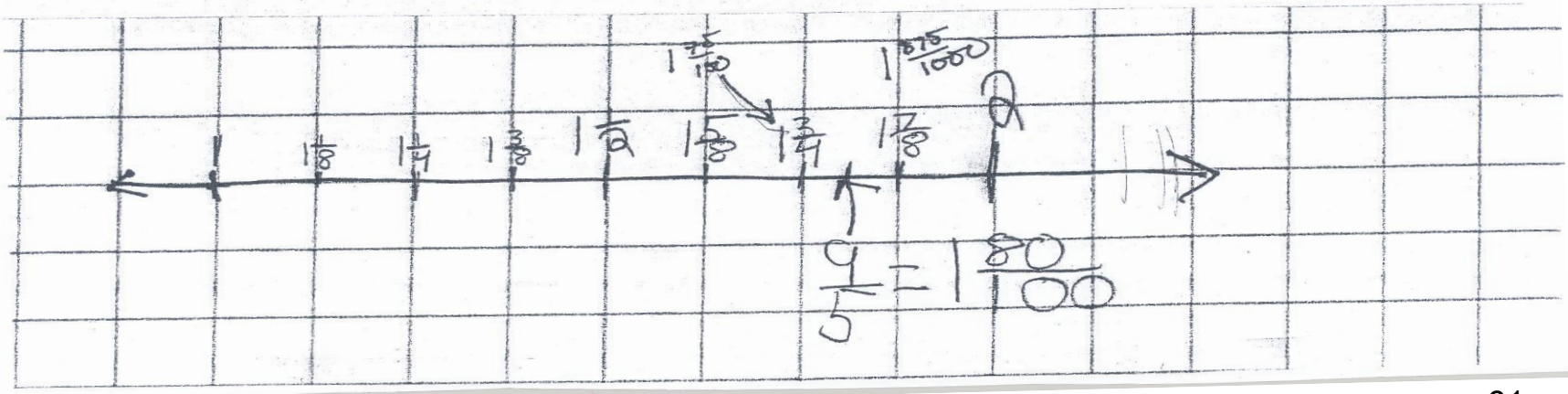
$\frac{9}{5}$ is equivlant to $1\frac{4}{5}$ and is almost 2 so it has to go there.

$\frac{7}{8} = \frac{87.5}{100}$

2) Order from smallest to greatest: 7/8, 3/8, 5/8, and 9/8.

$\frac{3}{8}$ $\frac{5}{8}$ $\frac{7}{8}$ $\frac{9}{8}$

$\frac{1}{8} = 12.5$



$\frac{9}{5} = 1\frac{80}{100}$

1) Draw a number line and show where to place the fraction 9/5. Explain your thinking.

Because 9/5 is a topheavy fraction I said it was = to 1 4/5 ($\frac{9}{5}$, 9-5=$\frac{4}{5}$ 1 4/5), 1 4/5 is right behind 2 on the # line.

2) Order from smallest to greatest: 7/8, 3/8, 5/8, and 9/8.

$\frac{3}{8}, \frac{5}{8}, \frac{7}{8}, \frac{9}{8}$

3) Order from smallest to greatest: 3/5, 3/7, 3/4, and 3/8.

$\frac{3}{8}, \frac{3}{7}, \frac{3}{5}, \frac{3}{4}$

$\frac{3}{4}$

- What happens to the value of the fraction if the numerator is increased by 1?
- What happens to the value of the fraction if the denominator is decreased by 1?
- What happens to the value of the fraction if the denominator is increased?

Ordering Fractions

Write these fractions in order from least to greatest. Tell how you decided.

- 5/3 5/6 5/5 5/4 5/8
- 7/8 2/8 10/8 3/8 1/8

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Recommendation 2

1. Using bar diagrams can you show 3/4 and 7/8? Which is greater? How do you know? (It's okay to use circular or rectangular regions or the number line or double number lines.)
2. Use any of the materials (perhaps all of them!) to represent the following equivalent fractions:
   - 1/2 = ?/6
   - 2/3 = ?/6
   - One additional equivalent pair
3. Place 1/8, 5/8, 7/8, and 9/8 on your number line. Which is the smallest fraction? The largest? How do you know?
   - **Extend**: Now place 2/3, 2/8, 2/2, 2/10, and 2/1 on your number line. Which fraction is smallest? Largest? How do you know?
4. If you could have any of the following amounts of your favorite food at a party, which would you rather have and why? Use drawings, bar diagrams, or other tools to help you in your thinking.
   0.5, 60%, 7/8, 1/2, 0.78 or 0.01

Recommendation 2

Think About

4. How can you represent 5 ⅔? Use any representation you like. How about 17/4? Use any representation you like.
   - **Extend**: Is one representation model easier to "see" than others?
5. If you wanted to establish that 0.9 > 7/8, how would you do that?
6. How could you establish that 12/4 = 24/8 = 6/2?
7. Using your number line, establish 0 and 1 and then 1/2 and continue to halve fractions until you run out of space. What's the point of all of this?
8. When do the materials show their limitations with equivalence?
9. Why is the number line described as "under-utilized" as a representation tool?
10. Connections to CCSS (Grades 3, 4, 5)

You can't make this stuff up!

- The weather reporter on WCRB (a Boston radio station) said there was a 30% chance of rain. The host of the show asked what that meant.
- The weather reporter said, ``It will rain on 30% of the state."
- ``What are the chances of getting wet if you are in that 30% of the state?"
- ``100%."

- I wanted to establish with my intermediate grade level teachers that in grades 4 and 5 multiplication of whole numbers is essentially done – that is, fluency is expected and the same thing is true for addition and subtraction of fractions. In some ways this is gong to thought of as 'business as usual,' but far too many of our school's students are just not there – particularly with understanding. So, I've got a couple of great Livescribe hits involving Willie. Just listen.

- Multiplication: http://www.livescribe.com/cgi-bin/WebObjects/LDApp.woa/wa/MLSOverviewPage?sid=bRWBJdf6RDDM

- Fractions: http://www.livescribe.com/cgi-bin/WebObjects/LDApp.woa/wa/MLSOverviewPage?sid=LQcqHZxzMVqK

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Continuing teacher needs?

A. Do they know this is going on? Awareness? Awareness+?

B. Professional Development
   - Language
   - Unpacking the content – scope and sequence

- Curriculum Materials
- Assessments

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Language…

- Grade 1
  - Students should apply the principle of ***transitivity of measurement*** to make indirect comparisons, but they need not use this technical term.
  - ***Right rectangular prisms***
  - ***Right circular cones***
  - ***Right circular cylinders***
- Grade 3
  - Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction a/b as the quantity formed by *a* parts of size 1/b.
- Grade 5
  - Make a line plot to display…
- Grade 6
  - Display numerical data in plots on a number line, including dot plots, histograms, and box plots

Unpacking Grade 4

- **4.OA Operations and Algebraic Thinking**
  - Use the four operations with whole numbers to solve problems
  - Gain familiarity with factors and multiples
  - Generate and analyze patterns
- **4.NBT Number and Operations in Base Ten**
  - Generalize place value understanding for multi-digit whole numbers
  - Use place value understanding and properties of operations to perform multi-digit arithmetic using the standard algorithm for addition and subtraction

Unpacking Grade 4 (cont.)

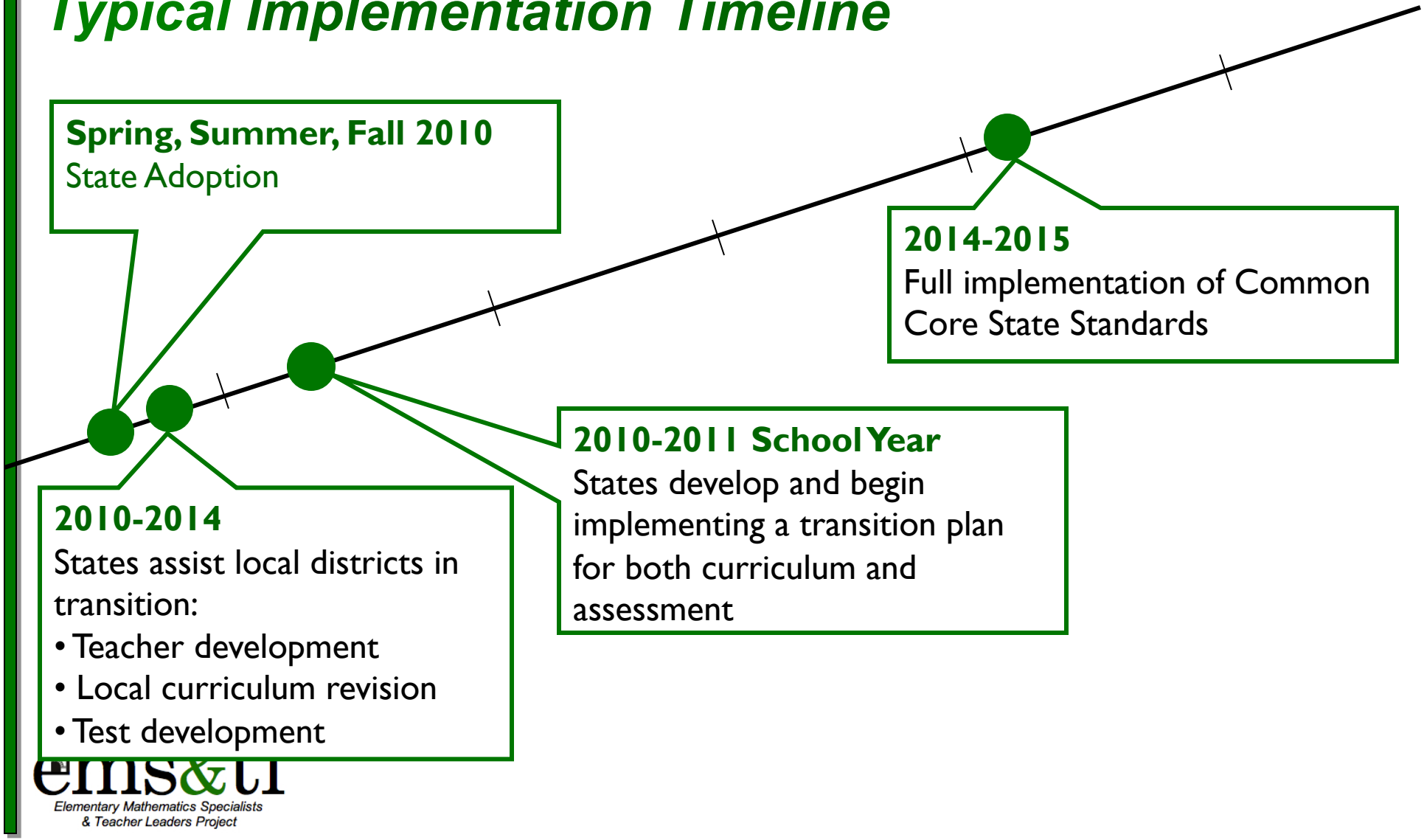
- **4.NF Number and Operations - Fractions**
  - Extend understanding of fraction equivalence and ordering.
  - Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
  - Understand decimal notation for fractions and compare decimal fractions.

Unpacking Grade 4 (cont.)

- **4.MD Measurement and Data**
  - Solve problems involving measurement and conversion of measurement from a larger unit to a smaller unit.
  - Represent and interpret data
  - Geometric measurement: understand concepts of angle and measure angles.
- **4.G Geometry**
  - Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Common Core State Standards

Typical Implementation Timeline



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What's going on…more

- Most states transitioning K-2…
- Promise of the *Mathematical Practices…*
- 2011-2012 – Grades K, 1, 2 (or portions)
- 2012-2013 – Grades 3-5
- 2013-2014 – Grades 6-8; Interim PARCC and Smarter Assessments
- 2014-2015 – PARCC and Smarter Assessments…

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Race to the Top Assessment Program Competition

- $350 million of Race to the Top Fund set aside for awards to consortia of states to design and develop common K-12 assessment systems aligned to common, college- and career-ready standards
- The competition asked consortia to design assessment systems that meet the dual needs of *accountability* and *instructional improvement*
- In September 2010, the U.S. Department of Education awarded grants to:
  - ***Partnership for Assessment of Readiness for College and Careers (PARCC)***
  - ***Smarter Balanced Assessment Consortium (SBAC)***
- The winning consortia have four years to develop assessments systems, and states participating in either consortium will administer new assessments statewide by 2014-2015

PARCC Assessments

- A mix of item types – short answer, longer open response and performance-based – in addition to richer multiple choice items that:
  - Better reflect the sophisticated knowledge and skills found in the English and math Common Core State Standards and
  - Will encourage teachers to focus on helping each student develop a deep understanding of the subject matter, rather than just narrowing their instruction in order to "teach to the test"
- Testing at key points throughout the year to give teachers, parents and students better information about whether students are "on track" or need some additional support in particular areas
- Recent design revisions, based on feedback from the PARCC states, rather than 4 "through-course" assessments; will create 2 summative assessments that could be used for accountability purposes as well as 2 optional assessment components – more formative. Draft materials available for review.

Taking your CCSS Pulse

- District level awareness AND plan
- Building level awareness and PLAN
- Teachers
  - Standards
  - Content; related language
- Materials
- Professional Development
- PARCC or SMARTER
  - awareness of plan

Pulse rate?

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Defining Adoption - Really

- 100% of the common core K-12 standards in mathematics to be adopted within 3 years
- Adoption of the common core either in its entirety or in its entirety with up to an additional 15% added ("85% rule").
- A state will have adopted when the **standards authorizing body within the state** has taken formal action to adopt and implement the common core.
- **States are responsible** for demonstrating that they have adhered to this definition of adoption.

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Role of US Dept. of Education here?

Implementation Resources…

- Progressions
- Illustrative Mathematics Project
- *Institute for Mathematics and Education – University of Arizona; Bill McCallum*

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Resources - Coming

- Articulating Research Ideas that Support the Implementation of the Professional Development Needed for Making the CCSS Reality – Karen Marrongelle, Peg Smith, Paola Sztajn – forthcoming report.
- COMAP – Curriculum and Assessment and the CCSS – on COMAP site.
- Development of a Research Agenda for Understanding the Influence of the Common Core State Standards in Mathematics – Horizon Research forthcoming, check Horizon site.
- Mathematics Curricular Analysis Tool – Bill Bush, University of Louisville, to be posted CCSS site.
- Math Forum – October 2-4, 2011 – CCSS and Teacher Education and Professional Development – Reston, VA
- MANY NCTM publications and opportunities – stay tuned and visit this very robust website regularly (existing "stuff" Curriculum Focal Points Grade and Grade Band Books, Essentials, and lots more).
- *NCTM, NCSM, AMTE, ASSM, CCSSO, PARC, SBAC – CCSS Coalition – look for*

We can't forget…

- RtI – defining tier needs with a CCSS curriculum
- Advanced students? Acceleration – particularly between elementary and middle school.

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Thanks for asking…

- Mathematical Practices
- K-2
  - What about PreK – Wisconsin, Ohio, NY
- What's Important
  - Focal Points and Critical Areas
- Beginning PD
  - Understanding and Representation
  - Scope and Unpacking
- Deciding Material (all of it) Needs

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Closing the Door on Innovation

Why One National Curriculum is Bad for America

A Critical Response to the Shanker Institute Manifesto and the U.S. Department of Education's Initiative to Develop a National Curriculum and National Assessments Based on National Standards

First, there is no constitutional or statutory basis for national standards, national assessments, or national curricula.

Be wary…Google conservative manifesto

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What are your steps toward transition and implementation?

Reality…

- No set of standards has much meaning without equitable resources to ensure that teachers are trained well enough to reach kids who live in widely different circumstances.
- …it is important to remember that neither these standards nor any other single effort will be the silver bullet some mistakenly believe is out there…

Valerie Strauss, June 7, 2010

Your Turn

Questions?

Want the slides?

ffennell.com

ffennell@mcdaniel.edu

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