Mathematics Teaching & Learning
Issues to Energize!

Francis (Skip) Fennell, President
National Council of Teachers of Mathematics
&
Professor of Education
McDaniel College
Westminster, MD

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Forces of Change

- Logical (Content - Mathematics)
- Psychological (Learning)
- Social (Parents, Society, Policy)

Ralph Tyler, 1934

Where have **we** been…

Recommended Actions

- The mathematics curriculum should be organized around **problem solving.**

- Mathematics teachers should create classroom environments in which **problem solving** can flourish.

An Agenda for Action. (Reston, VA: NCTM, **1980**, p. 2-5).

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Curriculum and Evaluation Standards

- Major Impact on K-12 Curriculum
- Influenced State Standards; 49 of 50 states now have state standards. - TEKS!!!
- Update: 30* states have revised their standards since 2003.
- Influenced Local School System Standards; most school districts now have their own standards.
- Grade Band Alignment – Grades K-4; 5-8; and 9-12 Standards
- **Note:** Prior to the Standards, the textbook was the defacto curriculum. It is far less likely that this is the case now – in the United States.

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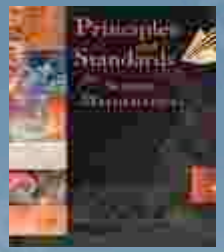
NCTM, 1989



Principles and Standards

Principles

- Equity
- Curriculum
- Teaching
- Learning
- Assessment
- Technology

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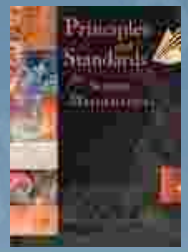
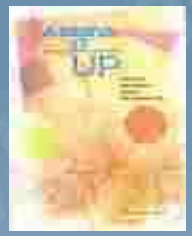


Content and Process Standards

- Number & Operations
- Algebra
- Geometry
- Measurement
- Data Analysis & Probability

- Problem Solving
- Reasoning & Proof
- Communication
- Connections
- Representation



Mathematical Proficiency

- **conceptual understanding** - comprehension of mathematical concepts, operations, and relations.
- **procedural fluency** - skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.
- **strategic competence** - ability to formulate, represent, and solve problems.
- **adaptive reasoning** - capacity for logical thought, reflection, explanation, and justification.
- **productive disposition*** - inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

The Happiness Factor?

Adding it Up, 2001

Oh my…

- "My wife who is completing her MS degree in Mathematics (a former HS Math teacher in the public schools), and I both agree, you are either born with a math gene or you're not (like many other god given abilities like performing arts, athletics, etc) and no amount of "confidence" or "fun" will convert most students into mathematicians, scientists, or engineers.

- The sooner we only focus on say the top 16-25% of students in math, the less frustrating math teachers will become."

Some Mathematics

What are they missing?

- If you are an elementary school teacher, what concepts, skills, and understandings do you hope students acquire **prior to leaving your grade level?**

- If you are a middle school teacher, what concepts, skills, and understandings do you wish most students would have when they come to you? **(but too many don't!)**

- If you are a high school teacher (of algebra), what concepts, skills, and understandings do you wish most students would have when they come to you? **(but too many don't!)**

- Finding and using patterns and other thinking strategies greatly simplifies the task of learning multiplication tables.

Thornton, 1978

- Children need to identify individual products rapidly. **Little is known about how children acquire this fluency or what experiences might be of most help.**

Adding it Up (NRC, 2001)

Think about…

- Teachers - In many cases, students should know their facts before they come to you. **OK, they don't - get over it!!**

- But, facts are important linchpins for upper level computation work.
- **How many times in your instructional life do you think you have dealt with this issue?**

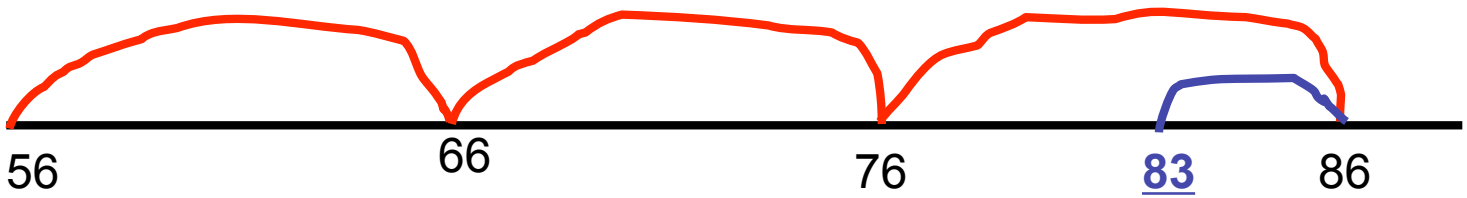
What can we make from:

- The numbers: 6 3 8

- Largest number?
- Smallest number?
- How many different 3-digit numbers?
- Sum in the 40's. Sum is? Addends are?
- Difference in the 50's. Difference is? Minuend? Subtrahend?

									0									
								1	2	3								
							4	5	6	7	8							
						9	10	11	12	13	14	15						
					16	17	18	19	20	21	22	23	24					
				25	26	27	28	29	30	31	32	33	34	35				
			36	37	38	39	40	41	42	43	44	45	46	47	48			
		49	50	51	52	53	54	55	56	57	58	59	60	61	62	63		
	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	
81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99

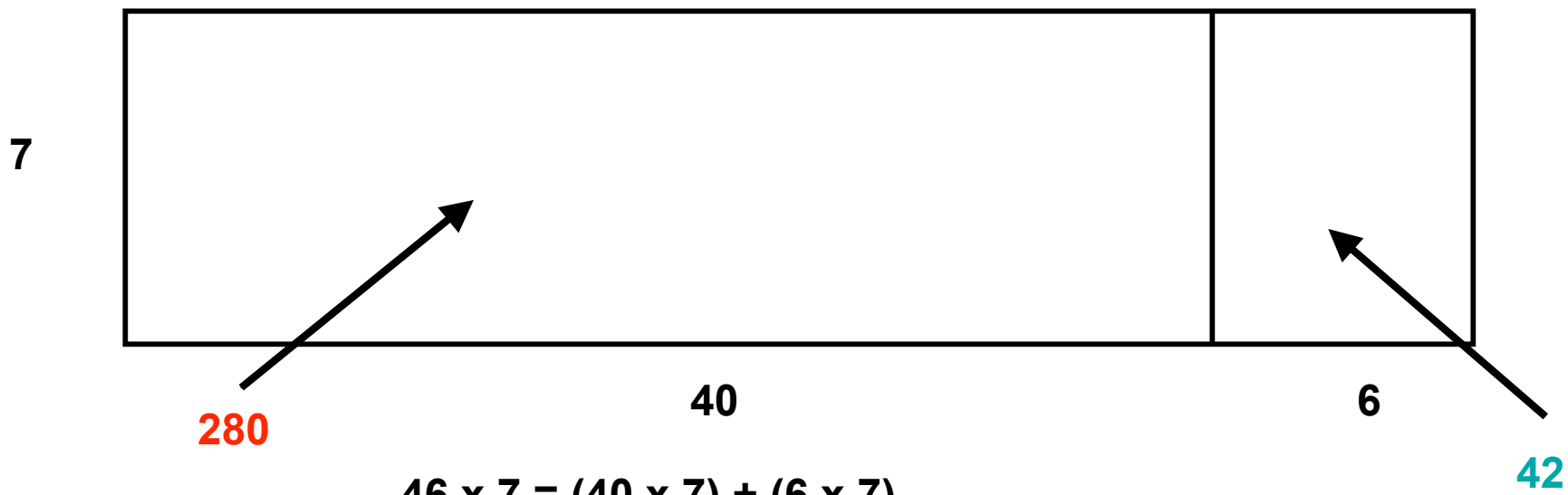
- Open number lines



56 + 27 =

Boxes to multiply…

- Draw a rectangle to show 46 x 7 = 322



46 x 7 = (40 x 7) + (6 x 7)
=

280 + 42 = 322

- **How about 45 x 23**

	40	5
20	40 x 20	5 x 20
3	40 x 3	5 x 3

$$\begin{array}{r} 45 \\ \underline{\times\ 23} \end{array}$$

Fractions

- A Mess! Children don't do well and never have.
- Links to number theory - GCF and LCM are not consistent across curricula.
- Social contexts for their use are diminishing, as we become a decimal culture.
- Links to decimals, ratio, percent, proportion.
- Elementary schools should begin the process and the **middle school should extend it.**

What's an improper fraction?

What did it do?

Comparing Fractions

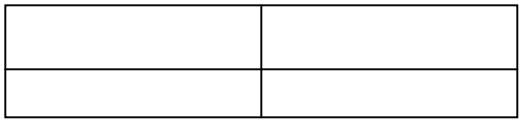
- Which is the greatest fraction? Tell how you know.

4/8	**4/7**	**4/6**	**4/9**
2/8	**4/8**	**1/8**	**3/8**

Write with numbers

¼ or 0.25

Draw a picture of it



Use words

One fourth; twenty five hundredths

Your own connection

A quarter is ¼ of a dollar

Units	Tenths	Hundredths
	1	3
1	3	

.13 x 10 = 1.3

Did we really move the decimal point?

Messy Problems are Good!

Imagine you are at your favorite amusement park, ready to ride the roller coaster. The ride lasts 2 minutes 15 seconds. The wait for the coaster each weekend is about 45 minutes, which is double the weekday wait.

- Could you ride the coaster three times in an hour?

Mathematical Discussions

- Procedural tasks for which students are expected to have well-developed algorithmic approaches are usually **not** good candidates for discussion. Interesting problems that "go somewhere" mathematically can often be catalysts for rich conversation.

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PSSM (NCTM, 2000, p. 60)

- Although in school, students are often presented with clearly specified problems to solve, outside of school they encounter situations in which part of the difficulty is to figure out exactly what the problem is. They then need to formulate the problem so that they can use mathematics to solve it.

Wiest, 2000.

Algebra

- Understand patterns, relations, and functions.
- Represent and analyze mathematical situations and structures using algebraic symbols.
- Use mathematical models to represent and understand quantitative relationships.
- Analyze change in various contexts.

Principles and Standards for School Mathematics (NCTM, 2000)

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Algebra: think abouts…

- The issue of algebra has become political.
- It's a gatekeeper
- It's very important for the serious study of collegiate mathematics
- Has become more of a K-12 issue.
- Issues:
  - **When?**
  - **Who?**
  - **What algebra?**
  - **Who's teaching?**
  - **Curriculum beyond algebra?**

Developing Algebraic Thinking

- The basic ideas of algebra as generalized arithmetic should be anticipated by activities in the early elementary grades and learned by the end of middle school.
- Teachers and researchers should investigate the effectiveness of instructional strategies in grades preK-8 that would help students move from arithmetic to algebraic ways of thinking.

Adding It Up, p. 419

Are you sure?

Actual problem presented at a mathematics conference.

A dog traveled 15 meters per second. How far would the dog go in: a minute, a half-hour, an hour, a day?

How much measurement? What measurement?

Speeds of some animals

Cheetah	70 mph
Lion	50 mph
Zebra	40 mph
Rabbit	35 mph
Reindeer	32 mph
Elephant	25 mph
Chicken	8 mph
"Super Dog"	32+ mph

The Mathematics

- Too many objectives, standards, etc.
- Every objective is not equally important across a grade or grade band.
- Ways to integrate and involve (daily!) the process standards.
- What are the big ideas or **focal points** of instruction?
- How can we get teachers to understand what is really important?

- How can we get teachers to teach, and teach well, topics in which they are not as familiar?

Number of 4th-Grade Learning Expectations per State by Content Strand

	Number & Operation	Geometry	Measurement	Algebra	Data Analysis, Probability & Statistics	Total Number of Learning Expectations
California	16	11	4	7	5	**43**
Texas	15	7	3	4	3	**32**
New York	27	8	10	5	6	**56**
Florida	31	11	17	10	20	**89**
Ohio	15	8	6	6	13	**48**
Michigan	37	5	11	0	3	**56**
New Jersey	21	10	8	6	11	**56**
North Carolina	14	3	2	3	4	**26**
Georgia	23	10	5	3	4	**45**
Virginia	17	8	11	2	3	**41**

Reys, et al., 2006

Curriculum Focal Points and Connections for Grade 4

The set of three curriculum focal points and related connections for mathematics in grade 4 follow. These topics are the recommended content emphases for this grade level. It is essential that these focal points be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

Grade 4 Curriculum Focal Points

***Number and Operations* and *Algebra:* Developing quick recall of multiplication facts and related division facts and fluency with whole number multiplication**

Students use understandings of multiplication to develop quick recall of the basic multiplication facts and related division facts. They apply their understanding of models for multiplication (i.e., equal-sized groups, arrays, area models, equal intervals on the number line), place value, and properties of operations (in particular, the distributive property) as they develop, discuss, and use efficient, accurate, and generalizable methods to multiply multidigit whole numbers. They select appropriate methods and apply them accurately to estimate products or calculate them mentally, depending on the context and numbers involved. They develop fluency with efficient procedures, including the standard algorithm, for multiplying whole numbers, understand why the procedures work (on the basis of place value and properties of operations), and use them to solve problems.

***Number and Operations:* Developing an understanding of decimals, including the connections between fractions and decimals**

Students understand decimal notation as an extension of the base-ten system of writing whole numbers that is useful for representing more numbers, including numbers between 0 and 1, between 1 and 2, and so on. Students relate their understanding of fractions to reading and writing decimals that are greater than or less than 1, identifying equivalent decimals, comparing and ordering decimals, and estimating decimal or fractional amounts in problem solving. They connect equivalent fractions and decimals by comparing models to symbols and locating equivalent symbols on the number line.

***Measurement:* Developing an understanding of area and determining the areas of two-dimensional shapes**

Students recognize area as an attribute of two-dimensional regions. They learn that they can quantify area by finding the total number of same-sized units of area that cover the shape without gaps or overlaps. They understand that a square that is 1 unit on a side is the standard unit for measuring area. They select appropriate units, strategies (e.g., decomposing shapes), and tools for solving problems that involve estimating or measuring area. Students connect area measure to the area model that they have used to represent multiplication, and they use this connection to justify the formula for the area of a rectangle.

Connections to the Focal Points

Algebra: Students continue identifying, describing, and extending numeric patterns involving all operations and nonnumeric growing or repeating patterns. Through these experiences, they develop an understanding of the use of a rule to describe a sequence of numbers or objects.

Geometry: Students extend their understanding of properties of two-dimensional shapes as they find the areas of polygons. They build on their earlier work with symmetry and congruence in grade 3 to encompass transformations, including those that produce line and rotational symmetry. By using transformations to design and analyze simple tilings and tessellations, students deepen their understanding of two-dimensional space.

Measurement: As part of understanding two-dimensional shapes, students measure and classify angles.

Data Analysis: Students continue to use tools from grade 3, solving problems by making frequency tables, bar graphs, picture graphs, and line plots. They apply their understanding of place value to develop and use stem-and-leaf plots.

Number and Operations: Building on their work in grade 3, students extend their understanding of place value and ways of representing numbers to 100,000 in various contexts. They use estimation in determining the relative sizes of amounts or distances. Students develop understandings of strategies for multidigit division by using models that represent division as the inverse of multiplication, as partitioning, or as successive subtraction. By working with decimals, students extend their ability to recognize equivalent fractions. Students' earlier work in grade 3 with models of fractions and multiplication and division facts supports their understanding of techniques for generating equivalent fractions and simplifying fractions.

Then What?

The Students and Learning

Gaps begin before children arrive at the schoolhouse door.

But, rather than organizing our educational system to ameliorate this problem, we organize it to exacerbate the problem.

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Education Trust, 2006

Adults Say:

- They're poor;
- Their parents don't care;
- They come to schools without breakfast;
- Not enough books
- Not enough parents . . .

OUR challenge!!!!!

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Education Trust, 2006

Time!!

Mathematics is the gateway to good jobs!

But it's not just Algebra II

They are all yours!

- There are 26 sheep and 10 goats on a ship. How old is the captain?

- What do you think middle grade students did with the problem above?

- More than three out of four students responded with a numeric answer, the most common one being that the captain's age is 36!
- One child explained: "Well, you need to add or subtract or multiply in problems like this, and this one seemed to work best if I add" (Bradford and Stein, 1993).

Finally, with regard to students and learning

There is nothing more unequal than equal treatment of unequal children.

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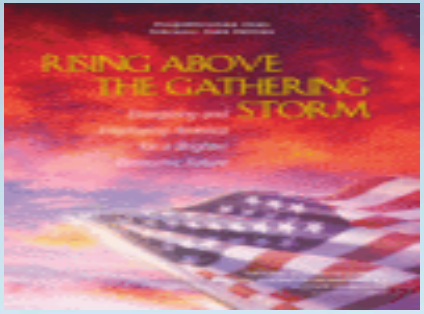
Policy/Community/Societal Issues

- 86% of U.S. voters believe that the U.S. must increase the number of workers with a background in science and mathematics or America's ability to compete in the global economy will be diminished.



- The Tom Friedman – effect!
- Competitiveness – an advantage for new ideas

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Rising Above the Gathering Storm, NAS, 2005

- ACT study notes gap between U.S. high school curriculum and college expectations.
  - Colleges generally want all incoming students to attain an in-depth understanding of a selected number of fundamental skills and concepts in their high school courses, while high schools tend to provide less in-depth instruction of a broader range of skills and topics.

April, 2007

More and better mathematics for all students

This is an imperative for America's competitiveness.

Do the kids care that Singapore is #1?

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Mixed Messages

- Nationwide (NAEP) Math Scores Improve for Grades 4 and 8!
- U.S. 15-year olds ranked 24th of 40 on the 2003 PISA examination – assessing the ability to solve real-world problems. New data coming.

- Is NAEP really the Nation's Report Card?
- NMP and NAEP

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NCTM (2007), PISA (2005)

No Child Left Behind Challenges

- Meeting AYP
- Highly Qualified Teachers
- Math for All
  - special Education challenges
  - the continuing use of the word "subgroups"
- Overemphasis on high stakes assessments.

These challenges are diverting teachers from teaching, and driving them from the profession!

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And, from my sister…*

Madaline Fennell, the Nebraska teacher of the year: Teachers are asking for,

- fully funding education and assessment programs that are federally mandated;
- language that addresses the special needs of students with disabilities, such as implementing state assessment systems that track the academic growth of individual students;
- replacing penalties against failing schools with methods to enhance achievement;
- multiple methods of assessment that evaluate a student's progress over the entire year, instead of just through standardized tests.

Ms. Fennell said that while there are positive aspects to the law, it is also "fraught with numerous deficiencies." The expertise of teachers who have been chosen as the best of the best in their states, she said, can help lawmakers craft a better version of the No Child Left Behind Act. "Teachers need to be included in this reauthorization," she said. "Please, leave no teacher behind."

* NOT

If we don't step up to the challenge of finding and supporting the best teachers we'll undermine everything else we are trying to do to improve our schools.

Louis Gerstner, former Chair, IBM

Technology and Instruction

We are digital immigrants and
our students are digital natives!

ACR National Policy Forum, 2005

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And of course, our need to support teachers…

Do you remember?
You were not prepared for your first class!
When did you really become a teacher?

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When I got everything done, I just sat down in the middle of my room and cried.

They never told me about:

- Getting along with others
- Finding stuff
- Faculty Lounge fallout
- Custodian
- Secretarial support
- Day-to-day victories and losses

SCHOOL CULTURE

"I didn't call myself anything. I was more than a teacher. And less. In the high school classroom you are a drill sergeant, a rabbi, a shoulder to cry on, a disciplinarian, a singer, a scholar, a clerk, a referee, a clown, a counselor, a dress-code enforcer, a conductor, an apologist, a philosopher, a collaborator, a tap dancer, a politician, a therapist, a fool, a traffic cop, a priest, a mother-father-sister-brother-aunt-uncle, a bookkeeper, a critic, a psychologist, the last straw."

"Teacher Man" Frank McCourt (p.19, 2005)

Thinking about teaching

- Teachers need expertise in both mathematics and in the teaching of mathematics.
- **Teachers are learners and the same principles of learning and transfer for student learners apply to teachers.**
- Teachers need opportunities to learn about children's cognitive development in order to know how teaching practices build on learners' prior knowledge.

Why do they leave?

- Too little planning time,
- Too much paperwork
- Unreliable assistance
- **A general lack of support, including limited pay!**

- And then: "There is a trust there (at my school). They look at me as a professional, and it really makes or breaks whether you stay."

Parents/Community/Societal/Policy Issues

Quiz

- Math is hard…
- Barbie!
- Math Sucks
- Jimmy Buffett
- You know I could never do math either…
- Every parent – seemingly!!!

Finally

- The challenges mathematics teachers face each day are many and varied in a job which is hard and seems to get harder each day. And you more than occasionally wonder who cares!!!

BUT…

ffennell@mcdaniel.edu
or
ffennell@nctm.org

http://ffennell.com

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